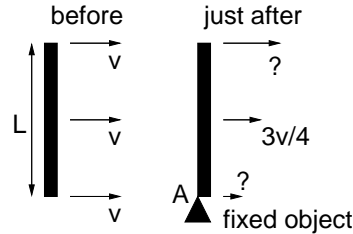


Problem 4: Collision (15 pts)

Initially, a rod of mass m and length L moves without rotation on a frictionless surface in a direction perpendicular to the rod. The speed of the rod is v . At time $t = 0$, one end of the rod collides with (or brushes over) a fixed object. Just after the collision, the rod is still parallel to the rod before the collision and the center of the rod still moves in the same direction as before. But the speed of the center of the rod is reduced to $\frac{3}{4}v$



- (a) Find the angular velocity ω of the rod just after the collision. (Hint: the total angular momentum about the collision point A is conserved during the collision.)
- (b) Find the total kinetic energy of the rod after the collision.
- (c) Find the speeds of the two ends of the rod just after the collision.

Solution:

(a) Angular momentum about point A:

Before collision: $mv\frac{L}{2}$

After collision: $m\frac{3v}{4}\frac{L}{2} + I_c\omega$.

(After collision, the angular momentum contains both contributions from the orbital motion of the center of mass and the spinning motion about the center of mass.)

$I_c = \frac{1}{12}mL^2$ is the moment of inertia about the center of mass of the rod.

From the conservation of the angular momentum $mv\frac{L}{2} = m\frac{3v}{4}\frac{L}{2} + I_c\omega$, we find the angular velocity $\omega = \frac{mv\frac{L}{2} - m\frac{3v}{4}\frac{L}{2}}{I_c} = \boxed{\frac{3v}{2L}}$.

(b) Before collision: $K = \boxed{\frac{1}{2}mv^2}$.

After collision: $K = \frac{1}{2}m\left(\frac{3v}{4}\right)^2 + \frac{1}{2}I_c\omega^2 = \boxed{\frac{3}{8}mv^2}$.

(c) Speed of the top end: $\frac{3}{4}v + \frac{L}{2}\omega = \boxed{\frac{3}{2}v}$.

Speed of the bottom end: $\frac{3}{4}v - \frac{L}{2}\omega = \boxed{0}$.