

~~Nov 12~~

~~Hour One: Problem Solving Session 17: Angular Momentum~~

~~Problem Set 10: Due Tues Nov 16 at 4:00 pm.~~

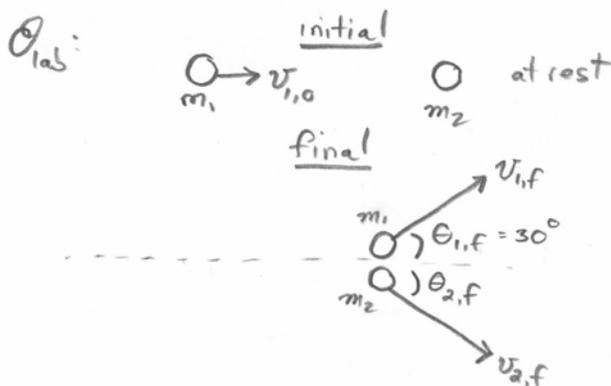
~~Problem 1: Impulse and Momentum~~

~~The compressive force per area necessary to break the tibia in the lower leg, is about $F/A = 1.6 \times 10^8 \text{ N/m}^2$. The smallest cross sectional area of the tibia, about 3.2 cm^2 , is slightly above the ankle. Suppose a person of mass $m = 6.0 \times 10^1 \text{ kg}$ jumps to the ground from a height $h_0 = 2.0 \text{ m}$ and absorbs the shock of hitting the ground by bending the knees. Assume that there is constant deceleration during the collision. During the collision, the person lowers his center of mass by an amount $\Delta d = 1 \text{ cm}$.~~

- ~~a) What is the collision time Δt_{col} ?~~
- ~~b) Find the average force of the ground on the person during the collision.~~
- ~~c) What is the average impulse of the ground on the person?~~
- ~~d) Will the person break his ankle? How much would you need to lower your center of mass so you do not break your ankle?~~

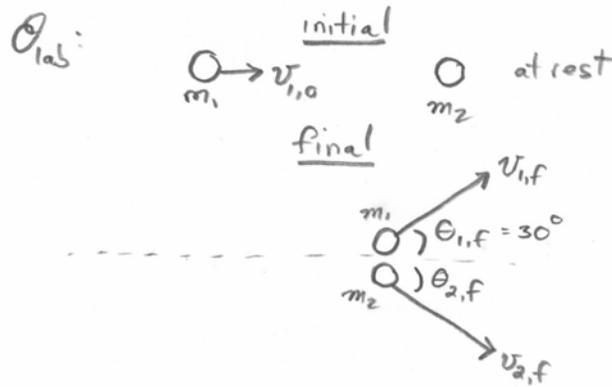
Problem 2: Two Dimensional Elastic Collision

An object with mass m_1 is initially moving with a velocity $v_{1,0} = 3.0 \text{ m/s}$ and collides elastically with another object of equal mass $m_2 = m_1$ that is initially at rest. After the collision m_1 moves with an unknown speed $v_{1,f}$ at an angle $\theta_{1,f} = 30^\circ$ with respect to its initial direction of motion. After the collision, m_2 moves with an unknown speed $v_{2,f}$, at an unknown angle $\theta_{2,f}$ (see sketch). Find the final velocities of each of the masses and the angle $\theta_{2,f}$.



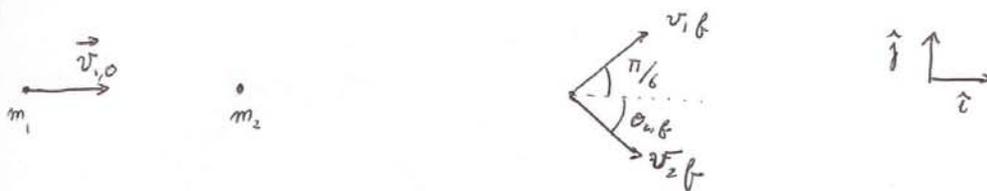
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Naturally we can solve the problem also entirely in the lab frame.

However this requires a good quantity of trigonometric alchemy:



$$\vec{P}_i = m_1 \vec{v}_{1,0} = m_1 |\vec{v}_{1,0}| \hat{i}$$

$$\vec{P}_f = m_1 \vec{v}_{1,f} + m_2 \vec{v}_{2,f} = m_1 (v_{1,f} \sin \theta_{1,f} + v_{2,f} \sin \theta_{2,f}) \hat{j} + m_1 (v_{1,f} \cos \theta_{1,f} + v_{2,f} \cos \theta_{2,f}) \hat{i}$$

$$\vec{P}_i = \vec{P}_f \Rightarrow \frac{1}{2} v_{1,0} = v_{2,f} \sin \theta_{2,f} \quad \text{from the } \hat{j} \text{ direction.}$$

$$v_{1,0} = \frac{\sqrt{3}}{2} v_{1,f} + v_{2,f} \cos \theta_{2,f} \quad \text{from the } \hat{i} \text{ direction.}$$

$$\begin{aligned} \Rightarrow v_{1,0} &= \sqrt{3} v_{2,f} \sin \theta_{2,f} + v_{2,f} \cos \theta_{2,f} = 2 v_{2,f} \left(\frac{\sqrt{3}}{2} \sin \theta_{2,f} + \frac{1}{2} \cos \theta_{2,f} \right) = \\ &= 2 v_{2,f} \sin \left(\theta_{2,f} + \frac{\pi}{6} \right) \quad v_{2,f} = \frac{v_{1,0}}{2 \sin \left(\theta_{2,f} + \frac{\pi}{6} \right)} \quad v_{1,f} = \frac{v_{1,0} \sin \theta_{2,f}}{\sin \left(\theta_{2,f} + \frac{\pi}{6} \right)} \end{aligned}$$

Then we can use conservation of energy to write

$$v_{1,0}^2 = \frac{v_{1,0}^2}{4 \sin^2 \left(\theta_{2,f} + \frac{\pi}{6} \right)} + \frac{v_{1,0}^2 \sin^2 \theta_{2,f}}{\sin^2 \left(\theta_{2,f} + \frac{\pi}{6} \right)} \Rightarrow 4 \sin^2 \left(\theta_{2,f} + \frac{\pi}{6} \right) = 1 + 4 \sin^2 \theta_{2,f}$$

$$\begin{aligned} 1 - \cos \left(2\theta_{2,f} + \frac{\pi}{3} \right) &= \frac{1}{2} + 1 - \cos(2\theta_{2,f}) \quad \cos(2\theta_{2,f}) - \cos \left(2\theta_{2,f} + \frac{\pi}{3} \right) = \frac{1}{2} \\ 2 \sin \left(2\theta_{2,f} + \frac{\pi}{6} \right) \sin \left(\frac{\pi}{6} \right) &= \frac{1}{2} \quad \sin \left(2\theta_{2,f} + \frac{\pi}{6} \right) = \frac{1}{2} \quad \theta_{2,f} = \frac{\pi}{3} \text{ or } \theta_{2,f} = 0 \end{aligned}$$

$\theta_{1,f} = 0$ represents the trivial solution (the two balls collide head on and the first stops and in fact $v_{1,f} = 0$ in this case)

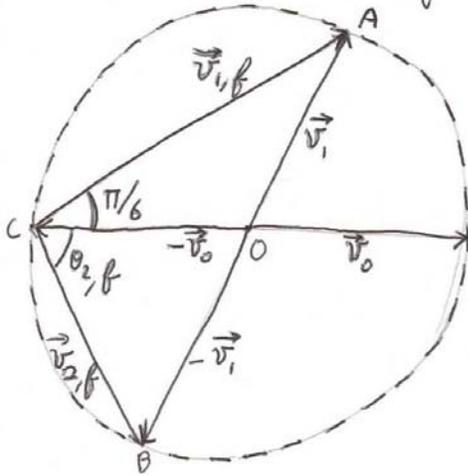
$\theta_{2,f} = \frac{\pi}{3}$ is more interesting, In this case.

$$v_{1,f} = \frac{v_{1,0}}{2} = 1.5 \text{ ms}^{-1} \quad | \quad v_{2,f} = v_{1,0} \frac{\sqrt{3}}{2} = 2.6 \text{ ms}^{-1} \quad \text{as before.}$$

Alternative solution in center of mass frame:

To solve the problem we will define $\vec{v}_0 = \frac{1}{2} \vec{v}_{1,0}$ the speed of the first ball in the c.m. frame, Then the second ball will have speed $-\vec{v}_0$. Setting the speed of the first ball after the collision in the c.m. frame as \vec{v}_1 we will have from momentum conservation that the second ball bounces with speed $-\vec{v}_1$. Then from energy conservation

$|\vec{v}_0| = v_0 = v_1 = |\vec{v}_1|$. The final speeds of the balls in the Lab frame (see figure) are: $\vec{v}_{1,f} = \vec{v}_1 - (-\vec{v}_0)$ $\vec{v}_{2,f} = -\vec{v}_1 - (-\vec{v}_0)$



Now $\frac{\pi}{6} + \theta_{2,f} = \frac{\pi}{2}$ because $\hat{A}BC = \frac{\pi}{2}$ as A, B, C lie on a circle of center O and $\hat{A}OB = \pi$

$$\theta_{2,f} = \frac{\pi}{3}$$

$$|\vec{v}_{1,f}| = 2 v_0 \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} v_{1,0} = 2.6 \text{ ms}^{-1}$$

$$|\vec{v}_{2,f}| = 2 v_0 \cos \frac{\pi}{3} = \frac{1}{2} v_{1,0} = 1.5 \text{ ms}^{-1}$$