Conservation of Momentum

8.01t

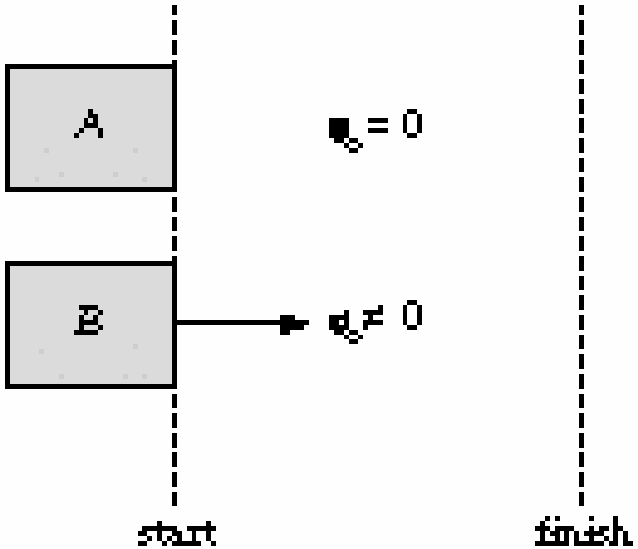
Oct 25, 2004

Momentum and Impulse

- momentum $$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$
- change in momentum $$\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}$$
- *average impulse* $$\vec{\mathbf{I}}_{ave} = \vec{\mathbf{F}}_{ave}\Delta t = \Delta\vec{\mathbf{p}}$$

PRS Question

Identical constant forces push two identical objects A and B continuously from a starting line to a finish line. If A is initially at rest and B is initially moving to the right,

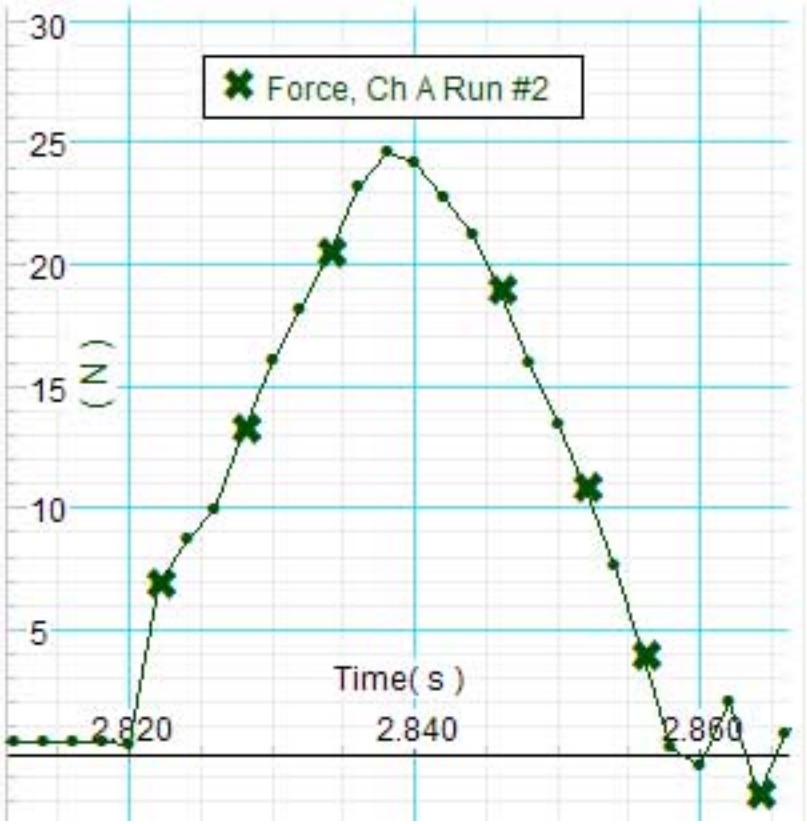


1. Object A has the larger change in momentum.
2. Object B has the larger change in momentum.
3. Both objects have the same change in momentum
4. Not enough information is given to decide.

Non-Constant Force and Impulse

- Cart colliding with a spring

$$\vec{\mathbf{I}} = \lim_{\Delta t_{i\to 0}} \sum_{i=1}^{i=N} \vec{\mathbf{F}}_i \Delta t_i \equiv \int_{t=0}^{t=t_f} \vec{\mathbf{F}}(t)dt$$



PRS Question

If a Mack truck and a Ford Escort have a head-on collision, which vehicle will experience the greater impulse?

1. the Mack truck
2. the Ford Escort
3. both experience the same impulse

PRS Question

If a Mack truck and a Ford Escort have a head-on collision, which vehicle will experience the greater acceleration?

1. the Mack truck
2. the Ford Escort
3. both experience the same acceleration

Conservation of Momentum

- *The total change in momentum of a system and its surroundings between the final state and the initial state is zero,*

$$\Delta\vec{\mathbf{p}}_{system} + \Delta\vec{\mathbf{p}}_{surroundings} = 0$$

External force on a System

- The total internal force on a system is zero

- Newton's Third Law: internal forces cancel in pairs

- total force acting on the system is the sum the external forces

$$\vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_{ext}^{total} + \vec{\mathbf{F}}_{\text{int}}^{total} = \vec{\mathbf{F}}_{ext}^{total}$$

External Force and Momentum Change

$$\vec{\mathbf{F}}^{total} = \sum_{i=1}^{i=N} \vec{\mathbf{F}}_i^{total} = \sum_{i=1}^{i=N} \frac{d\vec{\mathbf{p}}_i}{dt} = \frac{d\vec{\mathbf{p}}^{total}}{dt}$$

$$\vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_{ext}^{total}$$

$$\vec{\mathbf{F}}_{ext}^{total} = \frac{d\vec{\mathbf{p}}^{total}}{dt}$$

Center of Mass

- Position of center of mass

$$\vec{\mathbf{R}}_{cm} = \frac{\sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i}{\sum_{i=1}^{i=N} m_i}$$

- Velocity of center of mass

$$\vec{\mathbf{V}}_{cm} = \frac{\sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i}{\sum_{i=1}^{i=N} m_i} = \frac{\vec{\mathbf{p}}^{total}}{m^{total}}$$

Translational Motion of the Center of Mass

- Total momentum $\vec{\mathbf{p}}^{total} = m^{total}\vec{\mathbf{V}}_{cm}$

- External force and acceleration of center of mass

$$\vec{\mathbf{F}}_{ext}^{total} = \frac{d\vec{\mathbf{p}}^{total}}{dt} = m^{total}\frac{d\vec{\mathbf{V}}_{cm}}{dt} = m^{total}\vec{\mathbf{A}}_{cm}$$

Conservation of Momentum

- completely isolate system from the surroundings

$$\vec{\mathbf{F}}_{ext}^{total} = \vec{\mathbf{0}}$$

- change in momentum of the system is also zero

$$\Delta\vec{\mathbf{p}}_{system} = \vec{\mathbf{0}}$$

PRS Question

Drop a stone from the top of a high cliff. Consider the earth and a the stone as a system. As the stone falls, the momentum of the system

1. increases in the downward direction.
2. decreases in the downward direction.
3. stays the same.
4. not enough information to decide.

Modeling: External Forces and Conservation of Momentum

1. Identify the objects that compose the system

2. Identify your initial and final states of the system

3. Identify any external forces acting on the system

4. Decide whether momentum is conserved or not in each direction.

Modeling: Instantaneous Collisions

- Decide whether or not the collision is instantaneous.

- External impulse changes the momentum of the system.

$$\vec{\mathbf{I}}_{external} = \vec{\mathbf{F}}_{ext}\Delta t_{col} = \Delta\vec{\mathbf{p}}_{system}$$

- if the collision time is approximately zero, the momentum is approximately conserved.

- If $\Delta t_{col} \cong 0$ then $\Delta\vec{\mathbf{p}}_{system} \cong \vec{\mathbf{0}}$

Momentum Diagram

Draw a momentum diagram for the initial and final states.

a) Choose symbols to identify each mass and velocity in the system.

a) Identify a set of positive directions and unit vectors for each state.

c) Decide whether you are using components or magnitudes for your velocity symbols.

Momentum Diagram

Since momentum is a vector quantity, identify the initial and final vector components of the total momentum

Initial State $$\vec{\mathbf{p}}_0^{total} = m_1\vec{\mathbf{v}}_{1,0} + m_2\vec{\mathbf{v}}_{2,0} + \ldots$$

x-comp: $$p_{x,0}^{total} = m_1\left(v_x\right)_{1,0} + m_2\left(v_x\right)_{2,0} + \ldots$$

y-comp: $$p_{y,0}^{total} = m_1\left(v_y\right)_{1,0} + m_2\left(v_y\right)_{2,0} + \ldots$$

Final State $$\vec{\mathbf{p}}_f^{total} = m_1\vec{\mathbf{v}}_{1,f} + m_2\vec{\mathbf{v}}_{2,f} + \ldots$$

x-comp: $$p_{x,f}^{total} = m_1\left(v_x\right)_{1,f} + m_2\left(v_x\right)_{2,f} + \ldots$$

y-comp: $$p_{y,f}^{total} = m_1\left(v_y\right)_{1,f} + m_2\left(v_y\right)_{2,f} + \ldots$$

Conservation of Momentum

When the total external force on a system is zero, then the total initial momentum of the system equals the total final momentum of the system,

$$\vec{\mathbf{p}}_0^{total} = \vec{\mathbf{p}}_f^{total}$$

Strategies: Conservation of Momentum

- If momentum is conserved, write down the conservation equations for momentum in each direction

$$p_{x,0}^{total} = p_{x,f}^{total}$$

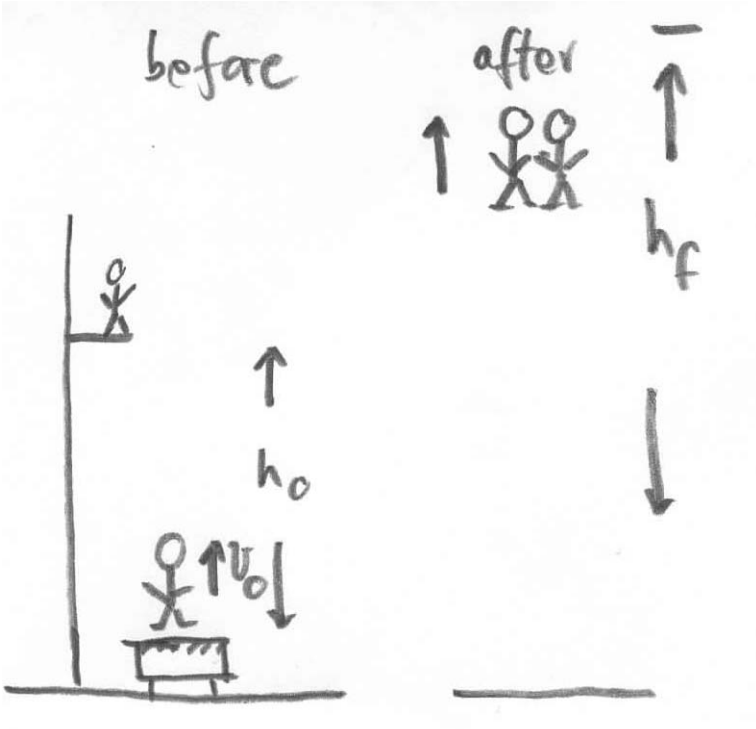
$$m_1\left(v_x\right)_{1,0} + m_2\left(v_x\right)_{2,0} + \ldots = m_1\left(v_x\right)_{1,f} + m_2\left(v_x\right)_{2,f} + \ldots$$

$$p_{y,0}^{total} = p_{y,f}^{total}$$

$$m_1\left(v_y\right)_{1,0} + m_2\left(v_y\right)_{2,0} + \ldots = m_1\left(v_y\right)_{1,f} + m_2\left(v_y\right)_{2,f} + \ldots$$

Class Problem 1: Acrobat and Clown

An acrobat of mass m_1 jumps upwards off a trampoline with an initial velocity v_0 . At a height h_0, the acrobat grabs a clown of mass m_2. Assume that the time the acrobat takes to grab the clown is negligibly small.



Problem 1: Acrobat and Clown (con't)

a) What is the velocity of the acrobat immediately before grabbing the clown?

b) What is the velocity of the acrobat immediately after grabbing the clown?

c) What high do the acrobat and clown rise?

d) If the acrobat had missed the clown, how high would have the acrobat risen?

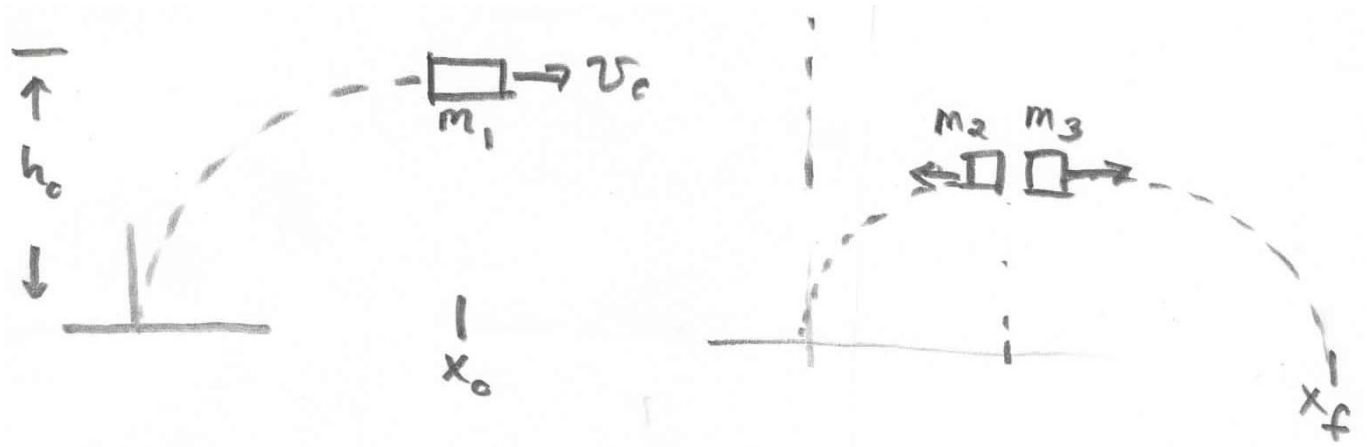
PRS Question

Suppose a golf ball is hurled at a heavy bowling ball initially at rest and bounces elastically from the bowling ball. After the collision,

1. The golf ball has the greater momentum and the greater kinetic energy.
2. The bowling ball has the greater momentum and the greater kinetic energy.
3. The golf ball has the greater momentum but has the smaller kinetic energy.
4. The bowling ball has the greater momentum but has the smaller kinetic energy.

Class Problem 2: Exploding Projectile

An instrument-carrying projectile of mass m_1 accidentally explodes at the top of its trajectory. The horizontal distance between launch point and the explosion is x_0. The projectile breaks into two pieces which fly apart horizontally. The larger piece, m_3, has three times the mass of the smaller piece, m_2. To the surprise of the scientist in charge, the smaller piece returns to earth at the launching station. Neglect air resistance and effects due to the earth's curvature.



Problem 2: Exploding Projectile (con't)

a) What is the velocity of the projectile at the top of its flight just before the collision?

b) What is the velocity of the smaller piece just after the collision?

c) What is the velocity of the larger piece just after the collision?

d) How far away does the larger piece land from the station?

PRS Question

Consider the exothermic reaction (final kinetic energy is greater than the initial kinetic energy).

$$H + H \rightarrow H_2 + 5ev$$

Two hydrogen atoms collide and produce a diatomic hydrogen molecule. Using only the principles of classical mechanics, this reaction

1. Is possible.
2. either violates conservation of energy or conservation of momentum but not both
3. satisfies conservation of energy and momentum but is not possible for other reasons.

Class Problem 3: Bouncing Superballs

- Two superballs are dropped from a height above the ground. The ball on top has a mass m_1 . The ball on the bottom has a mass m_2. Assume that the lower ball collides elastically with the ground. Then as the lower ball starts to move upward, it collides elastically with the upper ball that is still moving downwards. How high will the upper ball rebound in the air? Assume that $m_2 >> m_1$.

Hint: Problem 3: Bouncing Superballs

Consider this collision from an inertial reference frame that moves upward with the same speed as the lower ball has after it collides with ground. What speed does the upper ball have in this reference frame after it collides with the lower ball?