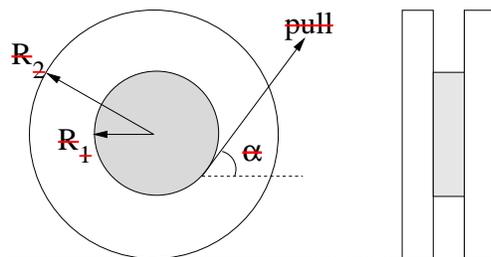


7.9 *A Classic! - Translation and Rotation PIVoT* — Look under “non-conservation of angular momentum” (you will also see some demonstrations there) and under “conservation of angular momentum”. A rod is lying at rest on a perfectly smooth horizontal surface (no friction). We give the rod a short impulse (a hit) perpendicular to the length direction of the rod at P. The mass of the rod is 3 kg, its length is 50 cm, the impulse is 4 kg·m/sec, the distance from the center C of the rod to P is 15 cm.

- What is the translational speed of C after the rod is hit?
- What is the angular velocity ω of the rod about C?
- What is the position of the rod 8 sec after it was hit; how far did C move, and what is the angle between the direction of the rod and its direction before it was hit?
- What is the total kinetic energy of the rod after it was hit?

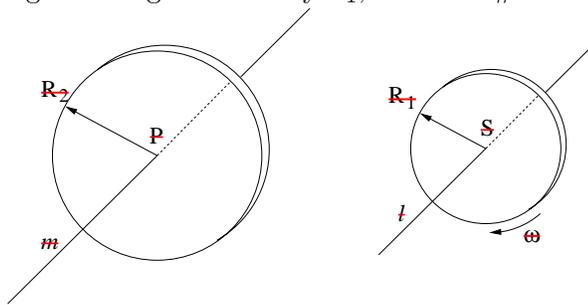
7.10 ~~The Amazing Yo-Yo! PIVoT~~

~~A yo-yo rests on the floor (the static friction coefficient with the floor is μ). The inner (shaded) portion of the yo-yo has a radius R_1 , the two outer disks have radii R_2 . A string is wrapped around the inner part. Someone pulls on the string at an angle α (see sketch). The “pull” is very gentle and carefully increased until the yo-yo starts to roll. Try it at Home; it’s Fun! You can watch the demo on PIVoT! For what angles of α will the yo-yo roll to the left and for what angles to the right?~~



7.11 ~~This is a difficult problem - It too is a Classic!~~

~~A solid disk with radius R_1 is spinning about a horizontal axle l at an angular velocity ω (it rotates freely; friction is ignored). The axle is perpendicular to the disk; it goes through the center S of the disk. The circumference of this disk (#1) is pushed against the circumference of another disk which is in all respects identical to #1 except that its radius is R_2 , and it is at rest. It can rotate freely about a horizontal axle, m , through P; m and l are parallel. The friction coefficient between the two touching surfaces (disk circumferences) is μ . We wait until an equilibrium situation is reached. At that time disk #1 is spinning with angular velocity ω_1 , and disk #2 with angular velocity ω_2 .~~



- ~~Is kinetic energy of rotation conserved? Give your reasons. Now imagine that you are doing this “experiment” and that you hold one axle m in your left hand and axle l in your right hand; you keep them parallel.~~
- ~~Do you have to apply a torque while you are pushing the disks against each other?~~
- ~~Is the total angular momentum of the two disks conserved?~~
- ~~Calculate ω_1 and ω_2 in terms of R_1 , R_2 , and ω . It is quite remarkable that ω_1 and ω_2 are independent of μ and independent of the time it takes for the equilibrium to be reached; i.e., independent of how hard one pushes the disks against each other.~~

$$\frac{1}{2}MR^2 \cdot \frac{a}{R} = RT \quad \implies \quad T = \frac{aM}{2}$$

~~Substituting this result into Equation (??) gives~~

$$ma = mg - \frac{aM}{2} \quad \implies \quad a = \frac{g}{1 + \frac{M}{2m}}$$

~~We can check that for $M \ll m$ that $a \rightarrow g$.~~

Problem 7.9

Choose the y axis parallel to the rod and the x axis along the direction of the hit, which is said to be perpendicular to the rod. Let $M = 3$ kg, $l = 50$ cm, and $d = 15$ cm.

(a) The impulse of the hit has magnitude $I = 4$ kg · m/s and direction \hat{x} , so let $\vec{I} = I\hat{x}$; this is given by

$$\vec{I} = \int \vec{F} dt = \int \frac{d\vec{P}}{dt} dt = \Delta\vec{P}$$

where \vec{P} is the total momentum of the system. The object is initially at rest, so the total momentum before the hit is zero. The total momentum after the hit, which is related to the center of mass velocity, is then

$$M\vec{v}_{CM} = \vec{P}_{after} = \Delta\vec{P} = \vec{I}$$

The point C is the center of the rod, which is the center of mass assuming the rod has uniform mass; therefore the translational speed of C is

$$|\vec{v}_{CM}| = \frac{|\vec{I}|}{M} = \frac{I}{M} = \frac{4}{3} \text{ m/s}$$

(b) If we assume that the hit occurs over a short enough time that the rod does not move appreciably, i.e. \vec{r} is constant, then

$$\int \vec{r} \times \vec{F} dt = \vec{r} \times \int \vec{F} dt = \vec{r} \times \vec{I}$$

The change in angular momentum is then given by

$$\Delta L = \int \frac{dL}{dt} dt = \int \vec{\tau} dt = \int \vec{r} \times \vec{F} dt = \vec{r} \times \vec{I}$$

The object is initially at rest, so the total angular momentum before the hit is zero, and the total angular momentum after the hit is

$$\vec{L} = \vec{r} \times \vec{I}$$

If we choose the origin of the coordinate system to coincide with C then the angular velocity about C is given by

$$I_{CM}\omega = |\vec{L}| = |\vec{r} \times \vec{I}| = dI$$

where $I_{CM} = \frac{1}{12}Ml^2$ is the moment of inertia through the center of the rod and perpendicular to the length of the rod. (Please see Table 12.1 on page 309.) Therefore

$$\omega = \frac{dI}{I_{CM}} = \frac{12 \cdot dI}{Ml^2} = 9.6 \text{ radian/s}$$

(c) After the hit there are no external forces, hence linear and angular momentum are conserved. The center of mass continues to move with the speed above; thus the distance it travels in 8 s is

$$D = v_{CM} \cdot 8 = \frac{4}{3} \cdot 8 \approx 11 \text{ m}$$

The rod also continues to rotate with the velocity above; thus the total angular rotation is

$$\Theta = \omega \cdot 8 = 76.8 \text{ radian}$$

This is clearly larger than 2π , so the angle between the direction of the rod before and after it is hit is given by

$$\theta = 76.8 - 12 \cdot 2\pi = 1.4 \text{ radian}$$

where $0 < 1.4 < 2\pi$ is the correct angle.

(d) The total kinetic energy of a rotating system is given by

$$K = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2 \approx 5.5 \text{ J}$$

Problem 7.10

~~Please see the sketch on the assignment.~~

~~Consider the direction to the right to be positive, and consider clockwise rotations to be positive. The force equation for horizontal motion is~~