

Review #3:
Collisions, Rotational Motion

Momentum - Model

From *Principia*: Momentum is defined as “the quantity of motion, conjointly proportional to the mass and the velocity”.

momentum is for a **system** of particles

definition ----
$$\mathbf{p} = \sum_{i=1}^{system} m_i \mathbf{v}_i$$

momentum is sum of individual momenta

Dynamics (change) of momentum - from $F=ma$

$$\frac{d\mathbf{p}^{system}}{dt} = \sum_{i=1}^{forces} \mathbf{F}_i^{external}$$

Only **external** (through system barrier) forces !

Strategies: Momentum and External Forces

1. Identify all forces acting on the masses
2. Select your system (make troublesome forces into internal forces; make $\Sigma F^{\text{ext}} = 0$)
3. $\Sigma F^{\text{ext}} = 0$ implies momentum is conserved
4. If there is a non-zero total external force:

$$\frac{d\mathbf{p}^{\text{system}}}{dt} = \sum_i^{\text{forces}} \mathbf{F}_i^{\text{external}}$$

Strategy - Impulse

Impulse most useful when time is short, simplifying the momentum change during the short time Δt .

A. One large external force dominates $\Delta \mathbf{p}$:

- Ball bouncing on floor - ignore gravity

B. Finite F^{ext} for very short $\Delta t \rightarrow \Delta \mathbf{p}^{\text{system}} = 0$

- Colliding cars - ignore horizontal friction

- Gun and Bullet - ignore external forces

Collisions: Momentum Conserved

When the total external force on the colliding particles is much smaller than the internal forces, the collision duration is so short that the impulse on the system is approximately zero.

Then the total initial momentum of the colliding particles equals their final momentum:

$$\mathbf{r}_{\mathbf{p}_f}^{total} = \mathbf{r}_{\mathbf{p}_0}^{total}$$

Gives one equation each for x, y, and z

Elastic Collisions

- Kinetic Energy does not change.

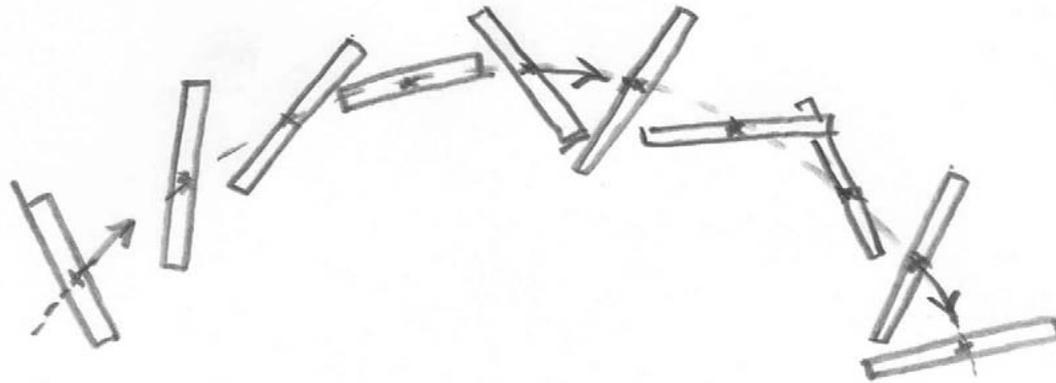
$$K_0^{total} = K_f^{total}$$

$$\frac{1}{2}m_1v_{1,0}^2 + \frac{1}{2}m_2v_{2,0}^2 + \dots = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2 + \dots$$

- Momentum conserved also

Rotation and Translation of Rigid Body

Motion of a thrown object



Translational Motion of the Center of Mass

- Total momentum $\mathbf{p}^{total} = m^{total} \mathbf{v}_{cm}$
- External force and acceleration of center of mass

$$\mathbf{F}_{ext}^{total} = \frac{d\mathbf{p}^{total}}{dt} = m^{total} \frac{d\mathbf{v}_{cm}}{dt} = m^{total} \mathbf{A}_{cm}$$