

**Problem 2: Conservation of Mechanical Energy and Newton's Second Law**

A small object of mass  $m = 0.2\text{ kg}$  is placed at the top of a large sphere of radius  $R = 0.5\text{ m}$  resting on the ground. The object is given a negligibly small velocity so that it starts to slide down the sphere. Assume the surface of the sphere is frictionless and the sphere is fixed to the surface of a table. In this problem, you will try to find where the object hits the ground.



- Briefly describe how you intend to model the motion. Here are some questions which should help guide your thinking. Is there any special condition(s) that describe when the object leaves the sphere? Does the normal force do any work on the object? Does the principle of Conservation of Energy replace Newton's Second Law in the radial direction or are they independent?
- At what angle will the mass leave the sphere?
- What is the velocity of the mass when it just leaves the sphere?
- How far from the bottom of the sphere will the mass hit the ground?

~~**Problem 3: Conservation of Energy and Newton's Second Law: Tetherball**~~

~~A ball of negligible radius and mass  $m = 0.1\text{ kg}$  hangs from a string of length  $l = 0.5\text{ m}$ . It is hit in such a way that it then travels in a vertical circle (with negligible loss of energy). The initial speed of the ball after being struck is  $v_0 = 7.0\text{ m/s}$ .~~



- ~~Find the speed of the ball at the top of the circle.~~
- ~~Find the tension in the string when the ball is at the top of the circle.~~

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- At what angle will the mass leave the sphere?
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a) The object leaves the sphere when the contact normal force between it and the sphere goes to 0.

After that the object is in free fall and we can apply the projectile motion rules. Before that we will use conservation of energy to find the speed of the object while sliding on the sphere. We will have to use Newton's second law to determine the normal force acting on the object from its acceleration.

b) First we have to compute the speed of the mass when it has slid for an angle  $\theta$  on the sphere. Using conservation of energy we find:

$$v(\theta) = \sqrt{2gR(1 - \cos(\theta))} = 2\sqrt{gR} \sin(\theta/2)$$

Then the radial component of the acceleration is:  $\vec{a} \cdot \hat{r} = -\frac{v^2}{R} = -4g \sin(\theta/2)$

From Newton's second law the normal force must be:  $|\vec{N}| = mg \cos(\theta) - 2gm(1 - \cos(\theta))$

notice that this equation is only valid before the mass leaves the sphere.

Imposing  $N = 0$  we find the angle at which the mass leaves the sphere:  $\theta = \arccos(2/3) = 0.27\pi$ . Notice that this point is at an height  $h = \frac{2}{3}R = 0.83 \text{ m}$  above the ground.

c) Using the previous formula for the speed we get  $|v| = \sqrt{\frac{2}{3}gR} = 1.8 \text{ m s}^{-1}$  this speed will have an horizontal component equal to  $|v_h| = 1.2 \text{ m s}^{-1}$  and a vertical component  $|v_v| = 1.34 \text{ m s}^{-1}$

d) After leaving the sphere the mass will fly for a time  $t$  such that:

$$\frac{1}{2}gt^2 + |v_v|t - h = 0 \text{ that is } t = g^{-1}(-|v_v| + \sqrt{|v_v|^2 + 2gh}) = 0.30 \text{ s}$$

In this time the mass covers the horizontal distance:  $d = |v_h|t = .36 \text{ m}$  and so touches the ground at a distance  $\frac{\sqrt{3}}{3}R + d = 0.73 \text{ m}$  from the bottom of the sphere.