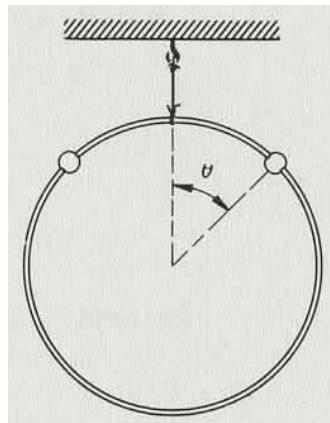


Problem 23: Beads Sliding along a Ring

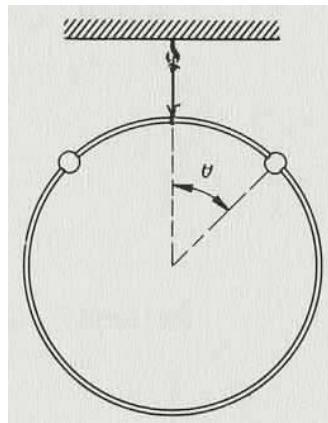
A ring of mass m_1 hangs from a thread, and two identical beads of mass m_2 slide on it without friction. The beads are released simultaneously from the top of the ring from rest (actually they need a very small initial speed but this can be ignored) and slide down opposite sides. Assume $m_2 > (3/2) m_1$. Then ring will start to rise when the beads reach a critical angle θ_c with respect to the vertical.



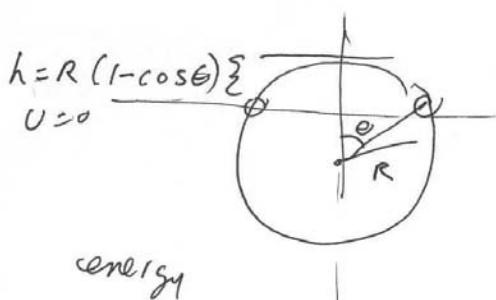
- a) Draw free body force diagrams for the ring and the beads. What direction is the force of the bead on the ring pointing? Does it change as the bead moves? Can you still proceed with an analysis using Newton's Second Law if you are not sure which way this force points? Try to find a physical explanation for the direction of this force. What is the condition that the ring just starts to rise?
- b) Does the mechanical energy change between when the beads are released and the ring just starts to rise? Write down an equation that describes the changes in the mechanical energy.
- c) **This is hard!** Can you show that the ring will start to rise if $m_2 > (3/2) m_1$? Find the angle θ_c with respect to the vertical direction that this occurs.

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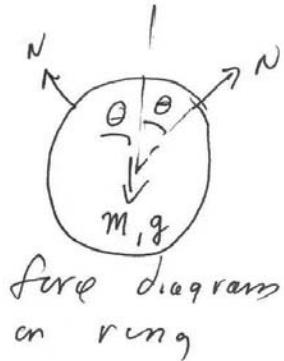
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- Does the mechanical energy change between when the beads are released and the ring just starts to rise? Write down an equation that describes the changes in the mechanical energy.
- This is hard!** Can you show that the ring will start to rise if $m_2 > (3/2) m_1$? Find the angle θ_c with respect to the vertical direction that this occurs.



energy

force diagram
on rungforce diagram
on bead

$$\vec{F}_1 = \begin{cases} m_1 \vec{a}_1 \\ 2N\cos\theta - m_1 g = 0 \end{cases} \Rightarrow N = \frac{m_1 g}{2\cos\theta} \quad (1)$$

condition that
the rung just
starts to rise

$$\vec{F}_2 = \begin{cases} m_2 \vec{a}_2 \\ -N - m_2 g \cos\theta = -\frac{m_2 v^2}{R} \end{cases} \Rightarrow N + m_2 g \cos\theta = \frac{m_2 v^2}{R} \quad (2)$$

$$\vec{E}_0 = \begin{cases} \vec{E}_f \\ R m_2 g (1 - \cos\theta) = \frac{1}{2} m_2 v^2 \end{cases} \quad \text{for singl6 bead} \quad (3)$$

Substituting eq (1) into eq (2) yields

$$\frac{m_1 g}{2\cos\theta} + m_2 g \cos\theta = \frac{m_2 v^2}{R} \quad (4)$$

Solving eq (3) for $m_2 v^2$ and substituting
into eq (4) yields

$$\frac{m_1 g}{2 \cos \theta} + m_2 g \cos \theta = 2m_2 g (1 - \cos \theta) \quad (5)$$

This simplifies to

$$m_1 g + 2m_2 g \cos^2 \theta = 4m_2 g \cos \theta - 4m_2 g \cos^2 \theta$$

or

$$m_1 g = 4m_2 g \cos \theta - 6m_2 g \cos^2 \theta$$

which is a quadratic eq for $\cos^2 \theta$

$$\cos^2 \theta - \frac{2}{3} \cos \theta + \frac{m_1}{6m_2} = 0 \quad (6)$$

solving

$$\cos \theta = \frac{2}{3} \pm \left(\frac{4}{9} - \frac{2}{3} \frac{m_1}{m_2} \right)^{1/2} \quad (6a)$$

when $m_1 = 0$ choose pos. root to get

$$\cos \theta = \frac{2}{3}$$

Suppose $\left(\frac{4}{9} - \frac{2}{3} \frac{m_1}{m_2} \right) > 0$ or we would

have an imaginary root

$$\frac{2}{3} > \frac{m_1}{m_2}$$

$$m_2 > \frac{3}{2} m_1 \quad (7)$$