

$$F = C_1 r v + C_2 r^2 v^2 \quad F = \frac{mMG}{r^2} \quad \mathbf{F} = \frac{d\mathbf{p}}{dt} = m\mathbf{a} \quad dW = \mathbf{F} \cdot d\mathbf{r} \quad a_{cent} = \frac{v^2}{r} = \omega^2 r$$

$$U = \frac{-mMG}{r} \quad U = mgh \quad U = \frac{1}{2} kx^2 \quad K = \frac{1}{2} m v^2 \quad K = \frac{1}{2} I \omega^2$$

$$E_{tot} = K + U = \frac{1}{2} m v^2 - \frac{mMG}{r} = \frac{-mMG}{2a} \quad \mathbf{L} = \mathbf{r} \times \mathbf{p} \quad I = \sum_i m_i r_i^2$$

$$m_1 r_1 = m_2 r_2 \quad v = \omega r \quad T^2 = \frac{4\pi^2 (r_1 + r_2)^3}{G(m_1 + m_2)} \quad \omega = \frac{d\theta}{dt} \quad \omega = \sqrt{k/m}$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = I\boldsymbol{\alpha} = \frac{d\mathbf{L}}{dt} \quad \alpha = \frac{d\omega}{dt} \quad T = \frac{2\pi}{\omega} \quad L = I\omega \quad \mathbf{I} = \int_0^{\Delta t} \mathbf{F} dt = \mathbf{p}_f - \mathbf{p}_i$$

$$\omega = \sqrt{g/l} \quad \omega_{pr} = \frac{\tau}{L_s} \quad T^2 = \frac{4\pi^2 a^3}{GM}$$

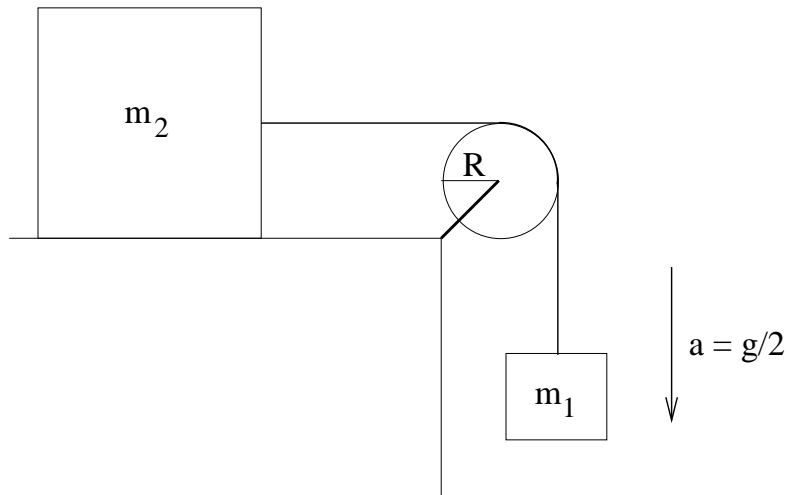
Solid disk of mass M and radius R rotating about its cylindrical axis: $I = \frac{1}{2} MR^2$

$$v_f - v_i = -u \ln \left(\frac{m_f}{m_i} \right) - gt \quad I = I_{cm} + Md^2 \quad I_z = I_x + I_y$$

$$f' = f \left(1 + \frac{v}{c} \cos \theta \right) \quad \lambda' = \lambda \left(1 - \frac{v}{c} \cos \theta \right)$$

Problem 1 (35 points)

An unknown mass, m_1 , hangs from a massless string and descends with an acceleration $g/2$. The other end is attached to a mass m_2 which slides on a frictionless horizontal table. The string goes over a uniform cylinder of mass $m_2/2$ and radius R (see figure). The cylinder rotates about a horizontal axis without friction and the string does not slip on the cylinder. Express your answers in parts b, c, and d in terms of g , m_2 , and R .

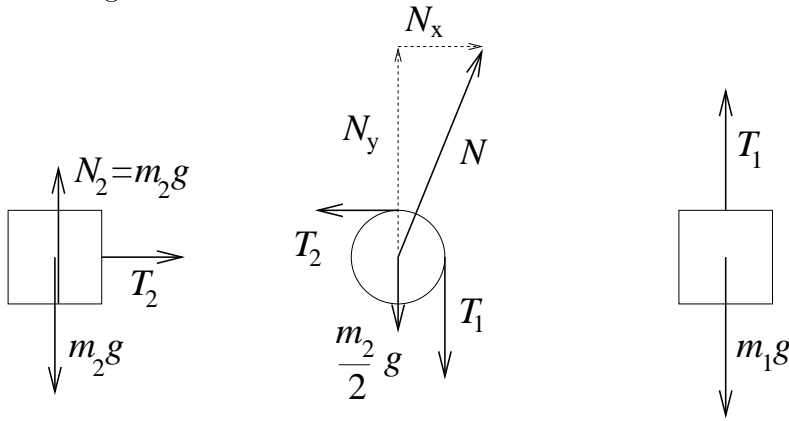


- a. (8) Draw free-body diagrams for the cylinder and the two masses.
- b. (9) What is the tension in the horizontal section of the string?
- c. (9) What is the tension in the vertical section of the string?
- d. (9) What is the value of the unknown mass m_1 ?

SOLUTIONS

Problem 1 35 points

8 pts a) The diagrams:



The contact force N on the pulley must have components $N_x = T_2$ and $N_y = \frac{m_2}{2}g + T_1$ for the pulley to stay in place.

9 pts b) $T_2 = m_2 a = m_2 \left(\frac{g}{2}\right)$

9 pts c) $R(T_1 - T_2) = I\alpha$ $I = \frac{1}{2} \frac{m_2}{2} R^2 = \frac{m_2 R^2}{4}$ $\alpha = \frac{a}{R}$
 $R(T_1 - m_2 \frac{g}{2}) = I \frac{a}{R} = \frac{I a}{R} = \frac{m_2 g R}{8}$
 $\Rightarrow T_1 = \frac{m_2 g}{8} + \frac{m_2 g}{2} = \frac{5}{8} m_2 g$

9 pts d) $m_1 g - T_1 = m_1 a = m_1 \frac{g}{2}$
 $m_1 \frac{g}{2} = T_1 = \frac{5}{8} m_2 g$
 $\Rightarrow m_1 = \frac{5}{4} m_2$