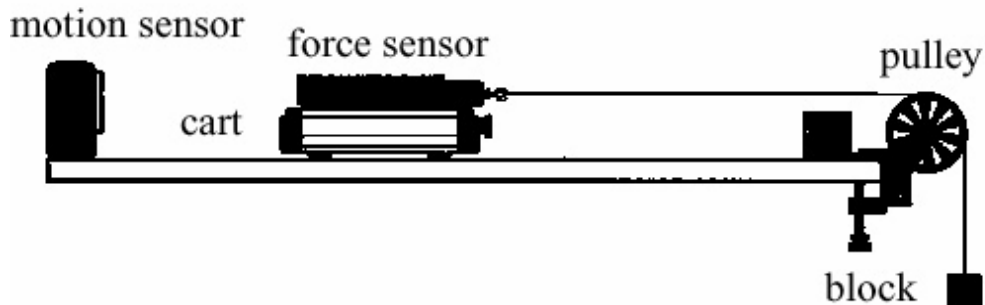


### In-Class Problem 5: Newton's Laws of Motion

Consider a track with a pulley located at the end. The force sensor and cart have total mass  $m_1$ . They are connected by an inextensible rope of length  $l$  (passing over the pulley) to a block of mass  $m_2$ . You may vary the mass of the block by adding additional weights. You may ignore the small mass of the rope and pulley. The coefficient of static friction between the car and the track is  $\mu_s$ , and the coefficient of kinetic friction is  $\mu_k$ . Assume that the coefficient of kinetic friction is less than the coefficient of static friction,  $\mu_k < \mu_s$ . Assume that the gravitational constant is  $g$ .



- What additional weight  $m_w$  do you need to add to the block such that the force sensor and cart just start slipping? Express your answer in terms of the relevant given information.
- Suppose you now add an extra weight  $m_3$  so that the total mass of the block and weights is  $m_2 + m_w + m_3$ . The coefficient of kinetic friction between the cart and the track is  $\mu_k$ . What then is the acceleration of the block?

You may want to consider these questions.

#### I. Understand – get a conceptual grasp of the problem

You may find the following helpful. Draw a coordinate system for both the cart and force sensor, and one for the block. Clearly indicate your choice of origins, axes, and choice of positive directions. Draw a (free body) force diagram for the system consisting of the cart and force sensor. Draw a (free body) force diagram for the block. Identify and reaction-action pairs of forces. Is the tension in the string constant?

#### II. Devise a Plan - set up a procedure to obtain the desired solution

What are the equations of motion for the system consisting of the cart and force sensor; and for the block? What is the “just slipping condition”? What physical condition is different in part b) compared to part a)?

**III. Carry out your plan – solve the problem!**

Do you have enough information to solve the problem?

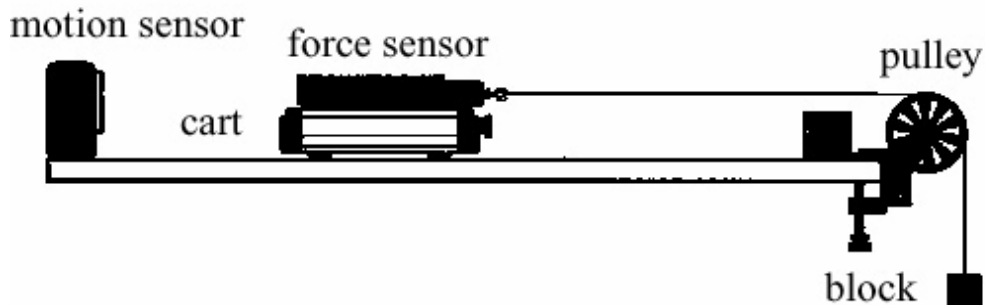
**IV. Look Back – check your solution and method of solution**

Do you think the coefficient of static friction is larger, equal to, or smaller, than the coefficient of kinetic friction? Explain your reasoning.

**Answer:**

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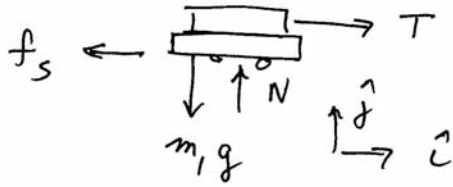
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Force Diagram  
on sensor and cart



$$\vec{F} = m \vec{a}$$

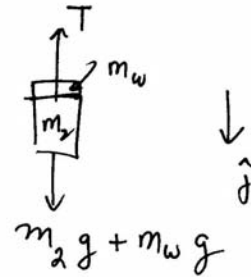

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$$\hat{i}: T - f_s = m_1 a_x$$

$$\hat{j}: N - m_1 g = 0$$

$$\Rightarrow N = m_1 g$$

Force Diagram on block:  
and extra weight



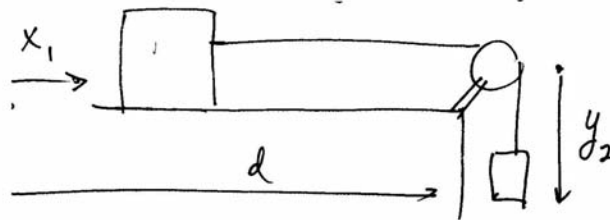
$$\vec{F} = m \vec{a}$$


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$$\hat{j}: (m_2 + m_w)g - T = (m_2 + m_w)a_y$$

Coordinate

system:



$$a \equiv a_x = a_y \text{ constraint}$$

Just slipping condition:  $a \equiv a_x = a_y = 0$

$$f_s = \mu_s N = \mu_s m_1 g$$

$$\left. \begin{array}{l} T - \mu_s m_1 g = 0 \\ (m_2 + m_w)g - T = 0 \end{array} \right\} \Rightarrow \begin{array}{l} (m_2 + m_w)g = \mu_s m_1 g \\ m_w = \mu_s m_1 - m_2 \end{array}$$

$$\vec{F}_1^T = m_1 \vec{a}$$

$$\begin{array}{l} \hat{i}: T' - f_k = m_1 a_x \\ \hat{j}: N - m_1 g = 0 \\ \quad N = m_1 g \end{array}$$

$$\vec{F}^T = m^T \vec{a}$$

$$(m_2 + m_w + m_3)g - T' = (m_2 + m_w + m_3)a_y$$

constraint  $a_x = a_y = a$ , string doesn't stretch

force law for kinetic friction:  $f_k = \mu_k N = \mu_k m_1 g$

$$T' - \mu_k m_1 g = m_1 a$$

$$(m_2 + m_w + m_3)g - T' = (m_2 + m_w + m_3)a$$

add equations:

$$(m_2 + m_w + m_3)g - \mu_k m_1 g = (m_1 + m_2 + m_3 + m_w)a$$

$$a = \frac{((m_2 + m_w + m_3) - \mu_k m_1)g}{(m_1 + m_2 + m_3 + m_w)}$$

where  $m_w = \mu_s m_1 - m_2$