

Problem 1: (Rotational kinematics)

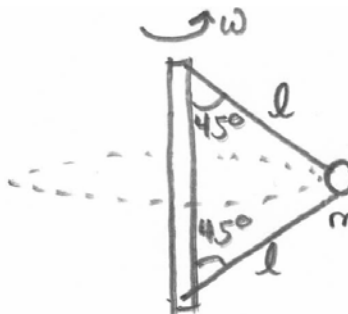
The earth is spinning about its axis with a period of 23 hours 56 min and 4 sec. The equatorial radius of the earth is 6.38×10^6 m. The latitude of Cambridge, Mass is $42^\circ 22'$.

- Find the velocity and centripetal acceleration of a person at MIT as they undergo circular motion about the earth's axis of rotation.
- The rotation of the Earth is slowing down. In 2004, the Earth took 1.01 s longer to complete 365 rotations than in 1924. What was the average angular deceleration of the Earth in the time interval from 1924 to 2004?

Problem 2: Second Law Applications: uniform circular motion

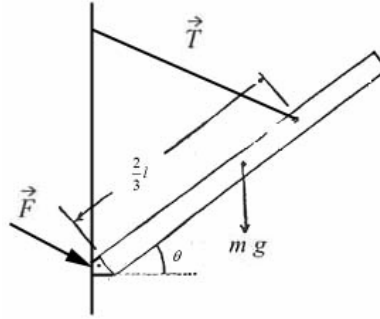
A mass m is connected to a vertical revolving axle by two massless inextensible strings of length l , each making an angle of 45° with the axle. Both the axle and the mass are revolving with angular velocity ω . Gravity is directed downwards.

- Draw a clear force diagram for m .
- Find the tension in the upper string, T_{upper} , and lower string, T_{lower} .



Problem 3: Experiment 5: Static Equilibrium Pre-Class Question

A pivoted beam is supported by a cable that is attached to the beam a distance $2/3$ of the way along the beam from the pivot. The center of mass of the beam is $1/2$ the distance from the pivot. The beam makes an angle θ with respect to the horizontal. There is a unknown force, \vec{F} , acting at the pivot.



If the beam is raised by tightening (and shortening the cable), explain whether the magnitude of force acting at the pivot, \vec{F} , increases, stays the same or decreases.

Problem 4: Experiment 04 Analysis

Fill in the table below with the numbers from your Experiment 04. Use at least 3 non-zero values of angular velocity ω , 4 if you made that many measurements. It is acceptable to do this problem in collaboration with your lab partners in the experiment. You may either do it in class using *DataStudio* or you may export your table from *DataStudio* as a text file and save it, and do the fit later using software such as *Xess*.

Data Table: experiment 04

ω					
r_m					

Fit these data to an expression of the form

$$r_m(\omega) = \frac{A}{1 - (\omega/\omega_c)^2}$$

Repeat the fit when A is not an adjustable parameter, but the value you measured for $r_0 = r_m(\omega = 0)$. When you do that, this is a fit with one adjustable parameter, and it is not necessary to include $\omega = 0$ in the fit; it is automatically included by the choice $r_0 = r_m(\omega = 0)$.

- What value did your fit give for A , and how does it compare to your measured r_0 ?
- Do you get a “better” fit when A is an adjustable parameter, or not? (Explain.)
- What did the one parameter fit give for the critical frequency ω_c ? Compare it to $\sqrt{k/m}$ for the spring in your experiment.
- What root mean squared error did the fit give you? How does it compare with the accuracy you estimate for your measurements of $r_m(\omega)$? (Remember, if you used a User-

Problem 1. Rotational Kinematics.

a) In 2004, the period of rotation is $T_E = 86164 \text{ s}$. Thus the angular velocity is

$$\omega_E = \frac{2\pi}{T_E} \approx 7.10 \cdot 10^{-5} \text{ s}^{-1}$$

The radius of the circle of rotation of the person at MIT is

$$r = R_E \cos \theta \approx 4.71 \cdot 10^6 \text{ m}.$$

Hence the linear velocity is

$$v = \omega_E r = \frac{2\pi}{T_E} R_E \cos \theta \approx 343.7 \text{ m/s}$$

and the centripetal acceleration is

$$a = \omega_E^2 r \approx 0.025 \text{ m/s}^2.$$

b) The average angular deceleration is

$$\left\langle \frac{d\omega_E}{dt} \right\rangle = \frac{\Delta\omega_E}{\Delta t} = \frac{2\pi}{T_{2004} T_{1924} \Delta t} (T_{2004} - T_{1924}) \approx \frac{2\pi}{T_{2004}^2 \Delta t} (T_{2004} - T_{1924})$$

where T_{2004} and T_{1924} refer to the period of the Earth's rotation in 2004 in 1924 and $\Delta t = 80$ years. Substituting numbers, we obtain

$$\left\langle \frac{d\omega_E}{dt} \right\rangle \approx 9 \cdot 10^{-22} \text{ s}^{-2}$$

Problem 2. Second Law Applications.

a) The force diagram is shown on Fig. 1.

b) Using the coordinate system shown on Fig. 1, we get

$$\begin{aligned} r : \quad T_1 \sin \theta + T_2 \sin \theta &= ma_r = m\omega^2 l \sin \theta \\ y : \quad T_1 \cos \theta - T_2 \cos \theta - mg &= 0, \end{aligned}$$

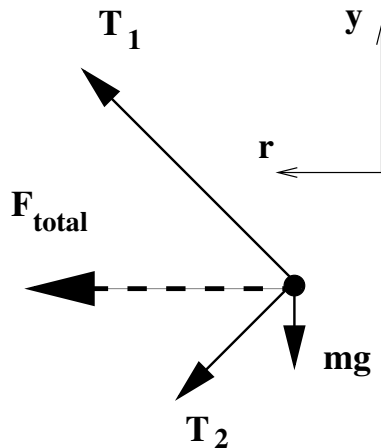


Figure 1: Force diagram for Problem 2

where $\theta = 45^\circ$ and $\sin \theta = \cos \theta = 1/\sqrt{2}$. Thus

$$T_1 = \frac{1}{2}m(\omega^2 l + \sqrt{2}g)$$

$$T_2 = \frac{1}{2}m(\omega^2 l - \sqrt{2}g)$$

Problem 3. Static Equilibrium.

When the beam is in equilibrium, the sum of all forces and the sum of all torques should be zero. Considering the torque about the pivot (see Fig. 2) and introducing the coordinate system with y pointing up and x pointing to the right, we have:

$$x : F_x - T \cos \phi = 0$$

$$y : F_y + T \sin \phi - mg = 0$$

and

$$\text{torque: } mg \frac{l}{2} \cos \theta - Th \cos \phi = 0$$

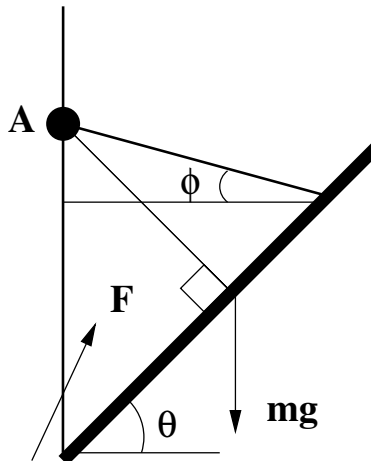


Figure 2: Beam supported by a cable

where T is the tension in rope, F_x and F_y denote x and y component of \vec{F} and we introduced two additional parameters — the height h of the point where the rope is connected to the pole and angle ϕ (see Fig. 2).

To solve the above equations, we note that the torque equation gives

$$T = mg \frac{l \cos \theta}{2h \cos \phi}$$

thus

$$F_x = mg \frac{l}{2h} \cos \theta$$

$$F_y = mg - mg \frac{l}{2h} \cos \theta \tan \phi.$$

From the geometry of the problem

$$\tan \phi = \frac{h - 2/3 l \sin \theta}{2/3 l \cos \theta}$$

Thus

$$F_y = mg \left(\frac{1}{4} + \frac{l}{2h} \sin \theta \right)$$

Thus the magnitude of the force is

$$F = \sqrt{F_x^2 + F_y^2} = \frac{mg}{4} \sqrt{1 + \frac{4l^2}{h^2} + \frac{4l}{h} \sin \theta}$$

and is increased when θ is increased.

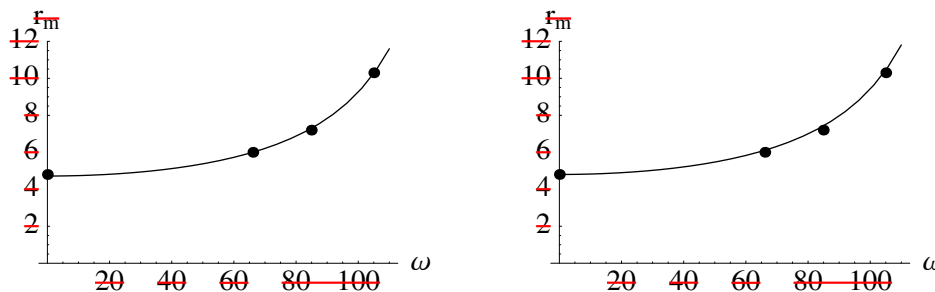
~~Problem 4. Experiment 4 Data Analysis.~~

~~We use the following data:~~

T	r_m	ω	ΔX	$m_m r_m \omega^2$
∞	4.8 cm	0 s^{-1}	0 m	0 N
0.095 s	6.0 cm	66.1 s^{-1}	0.012 m	2.44 N
0.074 s	7.2 cm	84.9 s^{-1}	0.024 m	4.83 N
0.060 s	10.3 cm	105 s^{-1}	0.055 m	10.5 N

~~a) Our fit gave $A = 4.7 \text{ cm}$, which is close (within the measurement error) to $r_0 = 4.8 \text{ cm}$.~~

~~b) We get a better fit when A is an adjustable parameter (root mean square error is $\sigma = 0.07 \text{ cm}$ vs $\sigma = 0.14 \text{ cm}$ when it is not adjustable — see Fig. 3).~~



~~Figure 3: The one and two parameter fits. Left: two-parameter fit. $A = 4.7 \text{ cm}$, $\omega_c = 143 \text{ s}^{-1}$, RMSE is 0.07 cm . Right: one parameter fit, $\omega_c = 144 \text{ s}^{-1}$, RMSE is 0.14 cm .~~

~~c) The one-parameter fit gives $\omega_c = 144 \text{ s}^{-1}$. On the other hand, $\sqrt{k/m} = 142 \text{ s}^{-1}$.~~