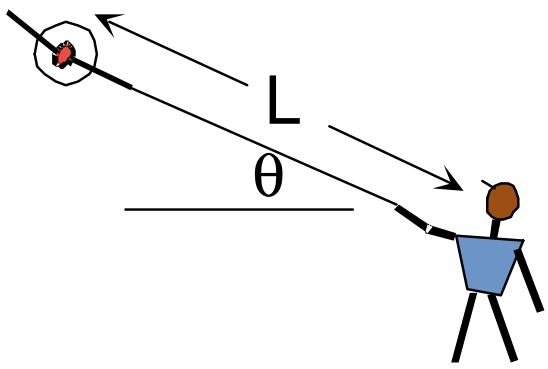
6. U-Control Model Airplane

A u-control airplane of mass M is attached by control wires of length L (and negligible mass) to the "pilot" who controls the lift provided by the wing. (The wires control the plane's elevator.) The plane's engine keeps it moving at constant speed v.

a. Find the total tension T in the wires when the plane is flying in a circle at an altitude such that the wires make an angle θ with the ground. Remember that the wings can provide lift only in the direction perpendicular to their area, i.e. in a direction perpendicular to the wires. Think carefully before selecting the angle of your coordinate system.
b. The plane will go out of control and crash if the tension is not maintained in the control wires. Given a particular speed of the plane, v, is there some angle θ_{crirt} which you would advise the pilot not to exceed?
c. If possible, exhibit a speed vsafe, at which the plane would be safe at any angle.



6) U-Control Model Airplane

Please refer to figure (3)

We choose the x axis along the acceleration so we have 0 acceleration along the y axis! If you choose the x axis along the wire you have have T along and F $\perp$. However you should project $\overrightarrow{\mathbf{a}}$ and elimination of two equations becomes difficult.

$$\sum \overrightarrow{\mathbf{F}} = m\overrightarrow{\mathbf{a}} \tag{47}$$

$$\sum F_y = ma_y = 0 \Rightarrow F\cos\theta - T\sin\theta - mg = 0 \tag{48}$$

$$\sum F_x = ma_x = \frac{mv^2}{R} = \frac{mv^2}{L\cos\theta} \Rightarrow F\sin\theta + T\cos\theta = \frac{mv^2}{L\cos\theta} \tag{49}$$

Eliminating F from (9) and (10) we get:

$$T - \frac{mv^2}{L} + mg\sin\theta = 0 \tag{50}$$

The physical condition T$\geq$ 0 will set a restriction on θ for a fixed v.

$$\sin\theta \leq \frac{v^2}{Lg} \tag{51}$$

So

$$\theta_{crit} = \arcsin\frac{v^2}{Lg} \tag{52}$$

Clearly for $v^2 = Lg$ even 90° will be fine. So there exists a finite v_{safe}:

$$v_{safe} = \sqrt{Lg} \tag{53}$$

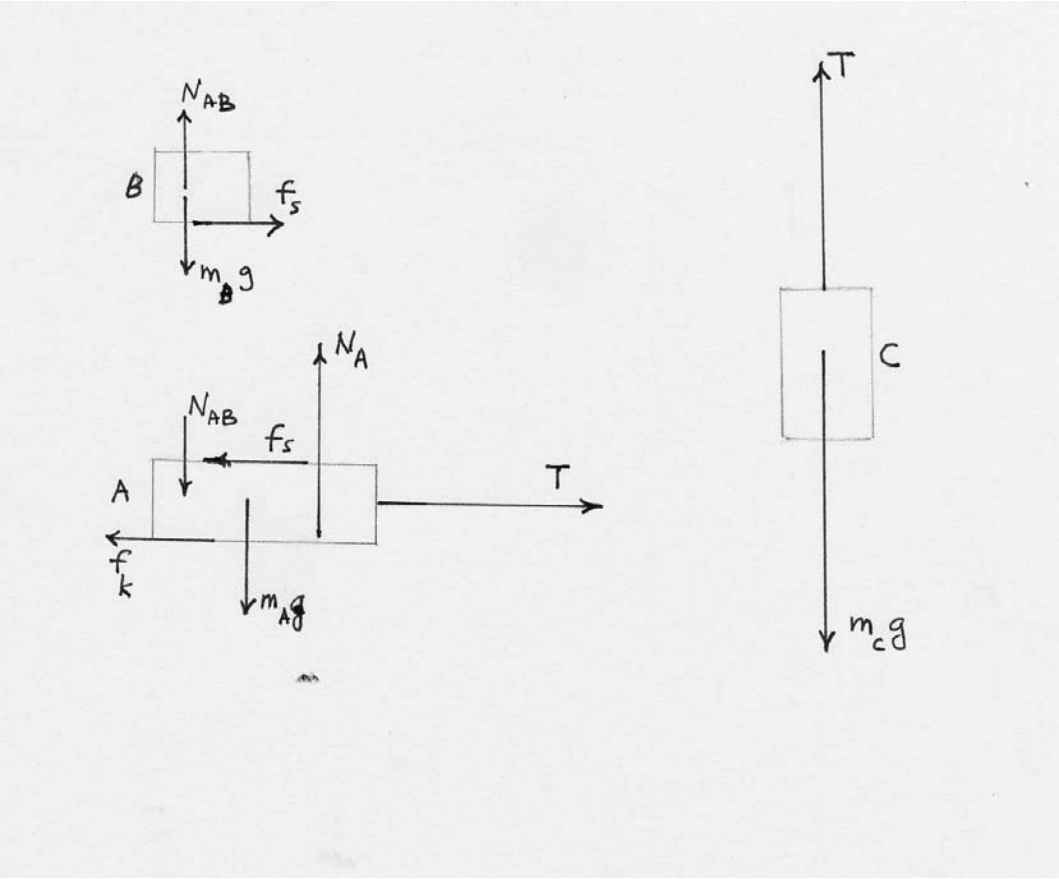


Figure 1: **5.86**

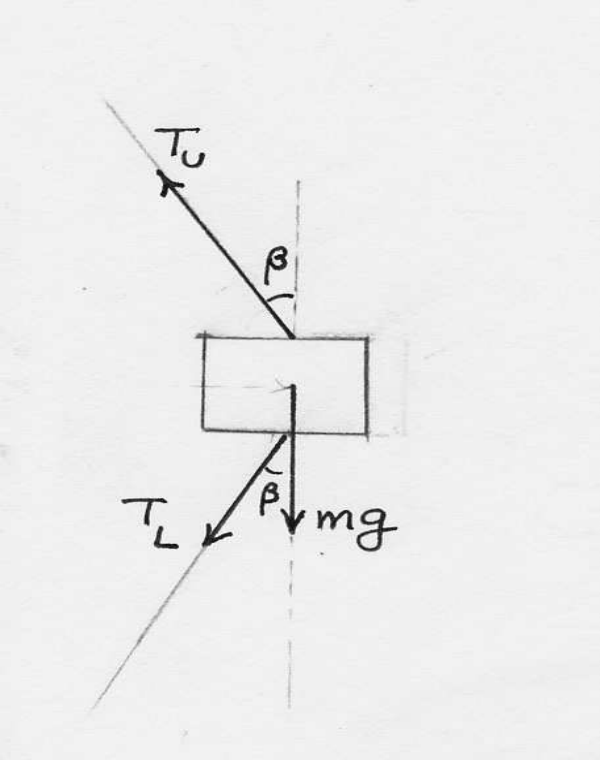


Figure 2: **5.104**

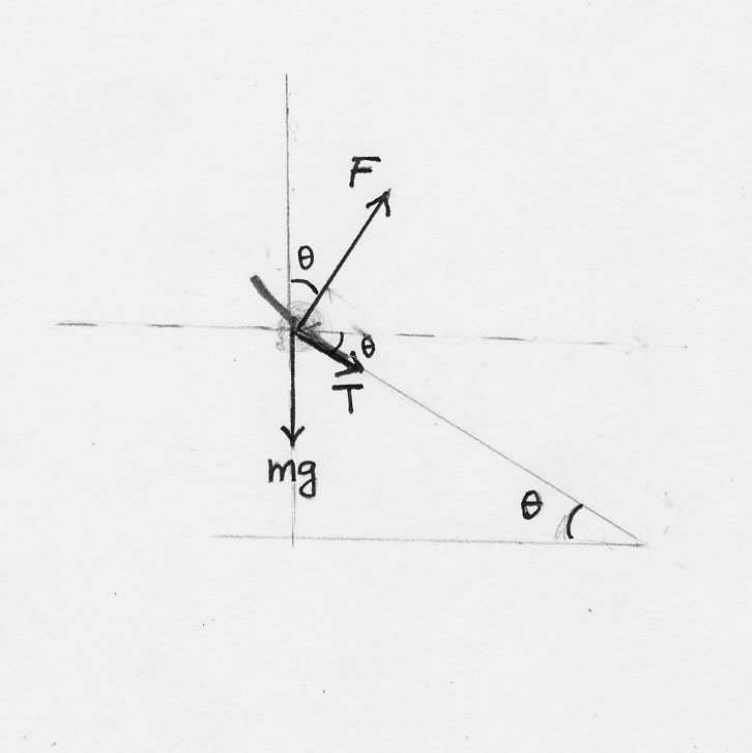


Figure 3: **U-Control Model Airplane**