

# **MASSAPEQUA PUBLIC SCHOOLS**

## **Common Core Algebra 2**

**Summer 2015**

### **COMMITTEE MEMBERS**

Lorajean DeFranco

Dahlia Evans

ToniAnne Summers, Curriculum Associate

### **BOARD OF EDUCATION**

Jane Ryan – President

Maryanne Fisher – Vice President

Gary Baldinger – Secretary

Tim Taylor – Trustee

Joseph LaBella – Trustee

### **ADMINISTRATION**

Lucille F. Iconis, Superintendent

Alan C. Adcock, Deputy Superintendent

Thomas Fasano, Ed.D., Assistant to the Superintendent for Curriculum & Instruction

Robert Schilling, Executive Director Assessment, Student Data and Technology Services

Diana Haanraadts, Asst. to the Superintendent for Instructional Support & General Administration

Dina Maggiacomo, Executive Director for Human Resources & General Administration

Jean Castelli, Executive Director of Special Education and Student Support Services

## **Course Description/Rationale**

The goal of this curriculum project is to create a curriculum that is aligned with the new Algebra II Common Core Curriculum. The Algebra II Common Core Curriculum is exceptionally ambitious and rigorous and is reflected in the depth and pace of the new curriculum.

Technology is integrated throughout the curriculum. Graphing calculators and when appropriate, dynamic software are used to investigate and/or explore many unit topics. In some lessons teachers will use the technology to demonstrate abstract concepts. At other times the students will use technology to explore lesson concepts either in groups or independently.

The curriculum was designed to support the district's Backwards Design and Differentiated Instruction initiatives. The curriculum identifies students' prior knowledge and important learning objectives.

Algebra II teachers will use this curriculum as they work together during their contractual professional periods to collaborate on lessons and department unit and review assessments.

Currently, there is no textbook.

## **Key Words for Curriki**

Common Core Algebra 2

## **Table of Contents**

[Course Description/Rationale](#)

[Key Words for Curriki](#)

[Common Core State Standards, Learning Standards, Key Ideas & Performance Indicators For Content Area](#)

[Curriculum Aligned with Common Core State Standards, Key Ideas & Performance Indicators](#)

[Unit 1: Operations with Polynomials](#)

[Unit 2: Real Zeros and Sketching Polynomial Functions](#)

[Unit 3: Modeling with Polynomials, Rational Expressions and Equations](#)

[Unit 4: Radical equations, Systems of Equations, Definition of Parabolas](#)

[Unit 5: Complex Numbers](#)

[Unit 6: Trigonometric Functions](#)

[Unit 7: Exponents, Exponential Equations, Logarithms, Log Equations](#)

[Unit 8: Exponential and Logarithmic Functions and their Graphs](#)

[Unit 9 : Sequences](#)

[Unit 10: Probability and Normal Distribution](#)

[Unit 11: Statistics](#)

**Common Core State Standards, Learning Standards, Key Ideas**

# & Performance Indicators For Content Area

## Module 1

N-Q.A.2.2 Define appropriate quantities for the purpose of descriptive modeling.

N-CN.A.1: Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.

N-CN.A.2: Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N-CN.C.7: Solve quadratic equations with real coefficients that have complex solutions.

A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .

A-APR.B.2: Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

A-APR.B.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-APR.C.4: Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.

A-APR.D.6: Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.A.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI.B.4: Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

A-REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .

F-IF.C.7: Graph functions expressed symbolically and show key features of the graph (by hand in simple cases and using technology for more complicated cases).

c. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.

G-GPE.A.2: Derive the equation of a parabola given a focus and directrix

N-CN.C.8: Extend polynomial identities to the complex numbers. For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .

N-CN.C.9: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Rewrite rational expressions.

A-APR.C.7: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

N-RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

N-Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret  $P(1 + r)^n$  as the product of  $P$  and a factor not depending on  $P$ .

A-SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

A-APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A-CED.A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3: Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A-CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning used in solving equations. For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .

A-REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.B.4: Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

A-REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

G-GPE.A.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

MP.1: Make sense of problems and persevere in solving them. Students discover the value of equating factored terms of a polynomial to zero as a means of solving equations involving polynomials. Students solve rational equations and simple radical equations, while considering the possibility of extraneous solutions and verifying each solution before drawing conclusions about the problem. Students solve systems of linear equations and linear and quadratic pairs in two variables. Further, students come to understand that the complex number system provides solutions to the equation  $x^2 + 1 = 0$  and higher-degree equations.

MP.2: Reason abstractly and quantitatively. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students also learn to make sense of remainders in polynomial long division problems.

MP.4: Model with mathematics. Students use primes to model encryption. Students transition between verbal, numerical, algebraic, and graphical thinking in analyzing applied polynomial problems. Students model a cross-section of a riverbed with a polynomial, estimate fluid flow with their algebraic model, and fit polynomials to data. Students model the locus of points at equal distance between a point (focus) and a line (directrix) discovering the parabola.

MP.7: Look for and make use of structure. Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. Students recognize structure in the graphs of polynomials in factored form and develop refined techniques for graphing. Students discern the structure of rational expressions by comparing to analogous arithmetic problems. Students perform geometric operations on parabolas to discover congruence and similarity.

MP.8: Look for and express regularity in repeated reasoning. Students understand that polynomials form a system analogous to the integers. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students recognize factors of expressions and develop factoring techniques. Further, students understand that all quadratics can be written as a product of linear factors in the complex realm.

## **Module 2**

F-IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.C.8: Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

S-ID.B.6a: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

F-TF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$  and use the unit circle to express the values of sine, cosine, and tangent for  $\pi - x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number.

F-TF.C.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. Foundational Standards Reason quantitatively and use units to solve problems.

N-Q.A.2: Define appropriate quantities for the purpose of descriptive modeling.

F-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is



the graph of the equation  $y=f(x)$ .

F-IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Build a function that models a relationship between two quantities.

F-BF.A.1: Write a function that describes a relationship between two quantities.

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- c. Compose functions. For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.

F-BF.B.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

G-SRT.C.6: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.C.7: Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

MP.1: Make sense of problems and persevere in solving them. Students look for entry points into studying the “height” of the sun above the ground, first by realizing that no such quantity exists, then by surmising that the notion can still be profitably analyzed in terms of trigonometric ratios. They use this and other concrete situations to extend concepts of trigonometry studied in previous years, which were initially limited to angles between zero and ninety-degrees, to the full range of inputs; they also solve challenges about circular motion.

MP.2: Reason abstractly and quantitatively. Students extend the study of trigonometry to the domain of all (or almost all) real inputs. By focusing only on the linear components of circular motion (the vertical or the horizontal displacement of a point in orbit), students develop the means to analyze periodic phenomena. Students also extend a classic proof of the Pythagorean theorem to discover

trigonometric addition formulas.

MP.3 Construct viable arguments and critique the reasoning of others. The vertical and horizontal displacements of a Ferris wheel passenger car are both periodic. Students conjecture how these functions are related to the trigonometric ratios they studied in geometry, making plausible arguments by modeling the Ferris wheel with a circle in the coordinate plane. Also, students construct valid arguments to extend trigonometric identities to the full range of inputs.

MP.4: Model with mathematics. The main modeling activity of this module is to analyze the vertical and horizontal displacement of a passenger car of a Ferris wheel. As they make assumptions and simplify the situation, they discover the need for sine and cosine functions to model the periodic motion using sinusoidal functions. Students then model a large number of other periodic phenomena by fitting sinusoidal functions to data given about tides, sound waves, and daylight hours; they then solve problems using those functions in the context of that data.

MP.7: Look for and make use of structure. Students recognize the periodic nature of a phenomenon and look for suitable values of midline, amplitude, and frequency for it. The periodicity and properties of cyclical motion shown in graphs helps students to recognize different trigonometric identities, and structure in standard proofs (of the Pythagorean theorem, for example) provides the means to extend familiar trigonometric results to a wider range of input values.

MP.8: Look for and express regularity in repeated reasoning. In repeatedly graphing different sinusoidal functions, students identify how parameters within the function give information about the amplitude, midline, and frequency of the function. They express this regularity in terms of a general formula for sinusoidal functions and use the formula to quickly write functions that model periodic data.

### **Module 3**

N-RN.A.: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define  $5^{\frac{1}{3}}$  to be the cube root of 5 because we want  $(5^{\frac{1}{3}})^3 = 5^{(\frac{1}{3})^3}$  to hold, so  $(5^{\frac{1}{3}})^3$  must equal 5.

N-RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-Q.A.2: Define appropriate quantities for the purpose of descriptive modeling.

A-SSE.B.34 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression  $1.15^t$  can be rewritten as  $(1.15^{\frac{1}{12}})^{12t} = 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A-SSE.B.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

A-CED.A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

F-IF.A.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1$ ,  $f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$ .

F-IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in

simple cases and using technology for more complicated cases.

- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y =$

$(1.01)^{12t}$ ,  $y = 1.2^{\frac{t}{10}}$ , and classify them as representing exponential growth or decay.

F-IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F-BF.A.1: Write a function that describes a relationship between two quantities.

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

F-BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F-BF.B.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF.B.4: Find inverse functions. a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$  or  $f(x) = (x + 1)/(x - 1)$  for  $x \neq 1$ .

F-LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.A.4: For exponential models, express as a logarithm the solution to  $ab^{ct}=d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

F-LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context

N-RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .

A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .

A-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y=f(x)$ .

F-IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Build a function that models a relationship between two quantities.

F-LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually

exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MP.1: Make sense of problems and persevere in solving them. Students make sense of rational and real number exponents and in doing so are able to apply exponential functions to solve problems involving exponential growth and decay for continuous domains such as time. They explore logarithms numerically and graphically to understand their meaning and how they can be used to solve exponential equations. Students have multiple opportunities to make connections between information presented graphically, numerically, and algebraically and search for similarities between these representations to further understand the underlying mathematical properties of exponents and logarithms. When presented with a wide variety of information related to financial planning, students make sense of the given information and use appropriate formulas to effectively plan for a long term budget and savings plan.

MP.2: Reason abstractly and quantitatively. Students consider appropriate units when exploring the properties of exponents for very large and very small numbers. They reason about quantities when solving a wide variety of problems that can be modeled using logarithms or exponential functions. Students relate the parameters in exponential expressions to the situations they model. They write and solve equations and then interpret their solutions within the context of a problem.

MP.4: Model with mathematics. Students use exponential functions to model situations involving exponential growth and decay. They model the number of digits needed to assign identifiers using logarithms. They model exponential growth using a simulation with collected data. The application of exponential functions and logarithms as a means to solve an exponential equation is a focus of several lessons that deal with financial literacy and planning a budget. Here, students must make sense of several different quantities and their relationships as they plan and prioritize for their future financial solvency.

MP.7: Look for and make use of structure. Students extend the laws of exponents for integer exponents to rational and real number exponents. They connect how these laws are related to the properties of logarithms and understand how to rearrange an exponential equation into logarithmic form. Students analyze the structure of exponential and logarithmic functions to understand how to sketch graphs and see how the properties relate to transformations of these types of functions. They analyze the structure of expressions to reveal properties such as recognizing when a function models exponential growth versus decay. Students use the structure of equations to understand how to identify an appropriate solution method.

MP.8: Look for and express regularity in repeated reasoning. Students discover the properties of logarithms and the meaning of a logarithm by investigating numeric examples. They develop formulas that involve exponentials and logarithms by extending patterns and examining tables and graphs.

Students generalize transformations of graphs of logarithmic functions by examining several different cases.

## **Module 4**

S-ID.A.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

S-IC.A.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S-IC.A.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

S-IC.B.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S-IC.B.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S-IC.B.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S-IC.B.6: Evaluate reports based on data.

S-CP.A.1: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

S-CP.A.2: Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP.A.3: Understand the conditional probability of A given B as  $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S-CP.A.4: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare results.

S-CP.A.5: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

S-CP.B.6: Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S-CP.B.7: Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

7.SP.B.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words



in a chapter of a fourth-grade science book.

7.SP.C.8: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
- c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

S-ID.A.2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID.A.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S-ID.B.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

MP.2: Reason abstractly and quantitatively. Students use data from a sample to estimate a population mean or proportion and generalize from a sample to the population. They associate a margin of error with estimates based on a sample and interpret them in the context of generalizing from a sample to the population. Students also make conjectures or claims about independence and use arguments based on probabilities to support them.

MP.3: Construct viable arguments and critique the reasoning of others. Students test conjectures about treatment differences in the context of a statistical experiment. Students critique and evaluate reports based on data from random samples and reports based on data from experiments. Students frequently develop conjectures and use statistical reasoning to evaluate them.

MP.4: Model with mathematics. Students use smooth curves to model data distributions. Students use the normal distribution as a model in order to answer questions about a data distribution.

Students use probability models to describe real-world contexts.

MP.5: Use appropriate tools strategically. Students use technology to carry out simulations in order to study sampling variability. Students also use technology to compute estimates of population characteristics (such as the mean and standard deviation) and to calculate margin of error. Students also use simulation to investigate statistical significance in the context of comparing treatments in a statistical experiment.

## **Curriculum Aligned with Common Core State Standards, Key Ideas & Performance Indicators**

Four mathematical practice standards are especially important in the Algebra II course:

**Construct viable arguments and critique the reasoning of others (MP.3).** As in geometry, there are central questions in advanced algebra that cannot be answered definitively by checking evidence. There are important results about *all* functions of a certain type — the factor theorem for polynomial functions, for example — and these require general arguments (A-APR.2). Deciding whether two functions are equal on an infinite set cannot be settled by looking at tables or graphs; it requires arguments of a different sort (F-IF.8).

**Attend to precision (MP.6).** As in the previous two courses, the habit of using precise language is not only a tool for effective communication but also a means for coming to understanding. For example, when investigating loan payments, if students can articulate something like, “What you owe at the end of a month is what you owed at the start of the month, plus  $1/12$  of the yearly interest on that amount, minus the monthly payment,” they are well along a path that will let them construct a recursively defined function for calculating loan payments (A-SSE.4).

**Look for and make use of structure (MP.7).** The structure theme in Algebra I centered on seeing and using the structure of algebraic expressions. This continues in Algebra II, where students delve deeper into transforming expressions in ways that reveal meaning. The example given in the standards — that  $x^4 - y^4$  can be seen as the difference of squares — is typical of this practice. This habit of seeing subexpressions as single entities will serve students well in areas such as trigonometry, where, for example, the factorization of  $x^4 - y^4$  described above can be used to show that the functions  $\cos 4x - \sin 4x$  and  $\cos 2x - \sin 2x$  are, in fact, equal (A-SSE.2).

In addition, the standards call for attention to the structural similarities between polynomials and integers (A-APR.1). The study of these similarities can be deepened in Algebra II: Like integers, polynomials have a division algorithm, and division of polynomials can be used to understand the factor theorem, transform rational expressions, help solve equations, and factor polynomials

**Look for and express regularity in repeated reasoning (MP.8).** Algebra II is where students can do a more complete analysis of sequences (F-IF.3), especially arithmetic and geometric sequences, and their associated series. Developing recursive formulas for sequences is facilitated by the practice of abstracting regularity for how you get from one term to the next and then giving a precise description of this process in algebraic symbols (F-BF.2). Technology can be a useful tool here: Most Computer Algebra Systems allow one to model recursive function definitions in notation that is close to standard mathematical notation. And spreadsheets make natural the process of taking successive differences and running totals (MP.5).

The same thinking — finding and articulating the rhythm in calculations — can help students analyze mortgage payments, and the ability to get a closed form for a geometric series lets them make a complete analysis of this topic. This practice is also a tool for using difference tables to find simple

functions that agree with a set of data.

Algebra II is a course in which students can learn some technical methods for performing algebraic calculations and transformations, but sense-making is still paramount (MP.1). For example, analyzing Heron's formula from geometry lets one connect the zeros of the expression to the degenerate triangles. As in Algebra I, the modeling practice is ubiquitous in Algebra II, enhanced by the inclusion of exponential and logarithmic functions as modeling tools (MP.4). Computer algebra systems provide students with a tool for modeling all kinds of phenomena, experimenting with algebraic objects (e.g., sequences of polynomials), and reducing the computational overhead needed to investigate many classical and useful areas of algebra (MP.5).

## **Unit 1: Operations with Polynomials**

## Student Outcomes:

- Students perform arithmetic operations on polynomials and write them in standard form
- Students understand the structure of polynomials expressions by quickly determining the first and last terms if the polynomial were to be written in standard form
- Students develop the distributive property for application to polynomial multiplication. Students connect multiplication of polynomials with multiplication of multi-digit numbers
- Students develop a division algorithm ( reverse tabular method) for polynomials by recognizing that division is the inverse operation of multiplication
- Students connect long division of polynomials with the long division of arithmetic and this algorithm to rewrite rational expressions that divide without a remainder
- Students work with polynomials with constant coefficients to prove polynomial identities
- Students understand that the sum of two square roots (or cubed roots) is not equal to the square root(or cube root) of their sum
- Students convert expressions to simplest radical form
- Students understand that the product of conjugate radicals can be viewed as the difference of two squares
- Students will factor certain forms of polynomial expressions by using the structure of the polynomials.

## Content map/calendar:

### Lesson 1: Adding, Subtracting, Multiplying Polynomials (2 days)

- Stress Properties that allow for each step in rewriting a polynomial expression

- Discuss closure property for the set of polynomials under the operations of addition, subtraction, and multiplication
- Multiply polynomials using tabular and non-tabular methods
- Generalize patterns that emerge during multiplication

### **Lesson 2: Dividing Polynomials (2 days)**

- Divide polynomials using reverse tabular method
- Extend patterns to make predictions about similar problems
- Divide polynomials using Long Division ( No remainders)

### **Lesson 3: Review and make Connections (1 day)**

- More complicated examples using two or more operations to write a polynomial in standard form
- Divide by factoring and removing forms of 1
- Determine the first and last terms **only** of a polynomial expression, if it were written in standard form

### **Lesson 4: Dividing by $x-a$ and $x+a$ (1 day)**

- Students recognize and then generalize patterns, and use them to fluently rewrite polynomial expressions and perform polynomial operations

\*Develops formulas for factoring sum and difference of cubes

- Use polynomial identities to perform arithmetic calculations ( if crunched for time put in

future do nows)

### **Lesson 6: Radicals and Conjugates ( 2 days)**

- Simplify square roots and cube roots - \* include variable radicands
- Add, subtract, multiply and divide radicals
- Find the conjugate of an irrational binomial
- Find Product of conjugates as difference of perfect squares - \*Stress Product of irrational conjugates is rational
- Rationalize irrational binomial denominator
- Stress vocab - rational, irrational, real

### **Lesson 7: Factor more complicated polynomials by analyzing their structure (1 day)**

- Use difference of squares identity to factor more advanced binomials
- Look for structure in trinomials where  $a \neq 1$  and apply previous factoring methods
- Review/practice factoring formulas for sum and difference of cubes

### **Lesson 8: Overcoming obstacles in factoring (1 days)**

- Factor by grouping
- Use completing the square technique to factor polynomials that have irrational factors

### **Technology Component:**

### **Assessment:**

Students will be given formal and informal assessments through the course as the end of the

unit.

## **Unit 2: Real Zeros and Sketching Polynomial Functions**

**Student Outcomes:**



- Students will factor certain forms of polynomial expressions by using the structure of the polynomials.
- Students find solutions to polynomial equations where the polynomial expression is not factored into linear factors.
- Students construct a polynomial function that has a specified set of zeros with stated multiplicity.
- Students will use the structure of polynomials to identify factors
- Students will use the factored forms of polynomials to find zeros of a function.
- Students will use the factored forms of polynomials to sketch the components of graphs between zeros.
- Students graph polynomial functions and describe end behavior based upon the degree of the polynomial.
- Students transition between verbal, numerical, algebraic, and graphical thinking in analyzing applied polynomial problems.
- Students rewrite simple rational expressions in different forms, including representing remainders when dividing.
- Students know and apply the Remainder Theorem and understand the role zeros play in the theorem.

### **Content map/calendar:**

#### **Lesson 1: The Special Role of zero in factoring (1 day)**

- Identify zeros of polynomials presented in factored form \* Just discuss factors that do not correspond to real zeros---solutions determined in complex number unit)

- Find real zeros by factoring polynomials
- Given a factor and how many real zeros - Find all zeros (use long division) and factoring
- Equate real zeros to x-intercepts on a graph
- Discuss multiplicity of zeros and how they relate to graph ( no discussion of complex zeros) even multiplicity vs odd multiplicity
- Write a polynomial function in factored form using the given zeros and their multiplicity- determine a possible degree of polynomial

**\* Real zeros only no sketching yet**

**\*Discuss factors that do not have real solutions**

## **Lesson 2: Graphing Polynomials (2 days)**

- Discuss multiplicity of zeros and how they relate to graph ( no discussion of complex zeros)
- Sketch Polynomial functions by finding zeros and testing for negative or positive values between between zeros---include graphs with roots of even or odd multiplicity
- Discuss Relative minimums and relative maximums
- Discuss intervals of increase or decrease
- Use Successive Differences to determine the possible degree of the function the table of values represents
- **Only do real zeros. Explain factors set = 0 like  $x^2+1=0$  not having a real solution so no x intercept on graph**

### **Lesson 3 : Structure in Graphs of Polynomial functions ( 2 days)**

- Compare graphs  $y = x^2$ ,  $y = x^4$ , and  $y = x^6$ . Discuss which increases faster, which is flatter
- Develop technique to determine the End Behavior of a polynomial function. Use end behavior to sketch polynomial functions
- Define Odd Functions and Even Functions Determine Odd or Even Functions Graphically and Algebraically

### **Lesson 4: Obstacle in factoring- Remainders (2 day)**

- Use reverse tabular method and long division to divide polynomials with remainders
- Develop method of inspection to divide polynomials with remainders
- Show (divisor)(quotient) + remainder = dividend

### **Lesson 5: The Remainder Theorem (2 days)**

- Develop the Remainder theorem
- Use Remainder Theorem to determine if a number is a zero of a polynomial
- Use Remainder theorem to determine if a given factor is a factor of a polynomial
- Given one zero find all zeros and sketch

### **Technology Component:**

Graphing calculator will be used to explore the behavior of polynomial functions.

### **Assessment:**

Students will be given formal and informal assessments through the course as the end of the unit.

## **Unit 3: Modeling with Polynomials, Rational Expressions and Equations**

### **Student Outcomes:**

- Students transition between verbal, numerical, algebraic, and graphical thinking in

analyzing applied polynomial problems

- Students learn to fit polynomial functions to data values
- Students model a cross-section of a riverbed with a polynomial function and estimate fluid flow with their algebraic model
- Students define rational expressions and write them in equivalent forms
- Students compare rational expressions by writing them in different but equivalent forms
- Students multiply and divide rational expressions and simplify using equivalent expressions
- Students perform addition and subtraction of rational expressions
- Students solve rational equations , monitoring for creation of extraneous roots
- Students solve word problems using models that involve rational expressions

### **Content map/calendar:**

#### **Lesson 1: Modeling With polynomials ( 2 days)**

- Use examples from module 1, lesson 16 and 17 to demonstrate the use of modeling real world problems with polynomial functions
- Use examples from module 1, lessons 20-21 to demonstrate the use of modeling real world problems with polynomial functions

#### **Lesson 2: Equivalent rational expressions and comparing rational expressions ( 1 days)**

- Discuss rational vs irrational numbers - extend to polynomials and rational expressions
- Define rational expressions - discuss division by zero undefined

- Reduce rational expressions by factoring - say equivalent for all values of variable except those that create zero in denominator
- Have students explain why 2 rational expressions are or are not equivalent
- Compare values of rational expressions by writing them in equivalent forms first

\* Avoid phrases like “cancel common factor” say “divide the common factor from the numerator and denominator”

### Lesson 3: **Multiplying and dividing rational expressions (1 day)**

- Multiply and divide by factoring
- Simplify complex fractions by creating an equivalent division problem

\* Division equivalent to multiplication of the numerator by the reciprocal of the denominator

\* Avoid phrases like “cancel common factor” say “divide the common factor from the numerator and denominator”

### Lesson 4: **Adding and Subtracting Rational Expressions (2 days)**

- Extend addition and subtraction of rational numbers to addition and subtraction of rational expressions

Find common multiple of denominators to use as common denominator

Find equivalent rational expressions for each expression using the common denominator

Add or Subtract the numerators and simplify if needed

### Lesson 5: **Solving Rational Equations (1 day)**

- Define extraneous solutions

- Show two methods to solve
  - Equating Numerators Method
  - Clearing Fractions Method

### **Lesson 6: Word Problems Leading to Rational Equations (1 day)**

- Develop student's problem solving skills
  - Carefully read a problem
  - Rephrase it in a form comfortable to their own understanding
  - Convert fact sentences about unknown quantities into algebraic equations

### **Technology Component:**

Graphing calculator will be used to explore the behavior of polynomial functions.

### **Assessment:**

Students will be given formal and informal assessments through the course as the end of the unit.

## Unit 4: Radical equations, Systems of Equations, Definition of Parabolas

### Student Outcomes:

- Students solve simple and complex radical equations and understand the possibility of extraneous solutions. They understand that care must be taken with the role of square roots so as to avoid apparent paradoxes
- Students explain and justify the steps taken in solving radical equations
- Students solve linear systems in three variables algebraically
- Students develop facility with graphical interpretations of systems of equations and the meaning of their solutions on those graphs.
- By completing the squares, students can convert the equation of a circle in general form to the center-radius form and, thus, find the radius and center. They can also convert the center-radius form to the general form by removing parentheses and combining like terms.
- Students understand how to solve and graph a system consisting of two quadratic equations in two variables.
- Students model the locus of points at equal distance between a point (focus) and a line (directrix). They construct a parabola and understand this geometric definition of the curve. They use algebraic techniques to derive the analytic equation of the parabola.
- Students learn the vertex form of the equation of a parabola and how it arises from the definition of a parabola.



- Students perform geometric operations, such as rotations, reflections, and translations, on arbitrary parabolas to discover standard representations for their congruence classes. In doing so, they learn that all parabolas with the same distance between the focus and the directrix are congruent to the graph of  $y = x^2$ .
- Students perform geometric transformations of dilation to show that all parabolas are similar

### **Content map/calendar:**

#### **Lesson 1: Solving Radical Equations (2 days)**

- Solve and check equations that contain one radical
- Solve and check equations that have 2 radicals and require isolating radical and squaring both sides twice
  - \* include square roots and cube roots
  - \* Compare operations with expressions containing radical vs equations

#### **Lesson 2: Solving Systems of Linear Equations in 2 Variables and systems of a linear and a quadratic equation in 2 variables GRAPHICALLY and ALGEBRAICALLY (2 days)**

- Have students make conjectures as to how many points of intersection there can be in a given system of equations
  - Use distance formula to find the distance between the centers of 2 circles to determine whether the circles intersect in 0, 1, or 2 points
- Review system of linear equations
- Sketch graphs of circle and a line- find point(s) of intersection

- include examples of completing the square to put equations of circles into center-radius form
- Sketch graphs of parabola and line - find point(s) of intersection
- Graphic solutions for 2 circles, 2 parabolas, circle-parabola
- Solve system of 2 linear, circle-linear, parabola-linear algebraically

\* Substitution and elimination methods----need linear system by elimination skills for 3x3 systems

\* Circle equations given in center-radius form as well as general form where completing square is required

\* Graphs of parabolas may be sketched using roots (found by factoring and/or calculator) and vertex point (found with calculator)

### **Lesson 3: Solving Systems of Equations in Three Variables (2 days)**

- Review Elimination Method to solve a system with 2 equations and 2 unknowns
- Develop method to Solve System of three equations with 3 unknowns

### **Lesson 4: Definition of a Parabola ( 1 day)**

- Model definition of parabola as the curve equidistant from a point(focus) and a line(directrix)
- Derive equation of parabola using algebraic techniques
  - Given focus and directrix
  - Given vertex

### **Lesson 5: Are Parabolas Congruent or Similar ( 1day)**

- Review writing/identifying vertex form of a parabola
- Develop an understanding as to when parabolas are congruent or similar

\*Use day to also review all transformations of functions

### **Technology Component:**

Graphing calculator will be used to explore the behavior of polynomial functions.

### **Assessment:**

Students will be given formal and informal assessments through the course as the end of the unit.

## Unit 5: Complex Numbers

### Student Outcomes:

- Students understand the possibility that an equation—or a system of equations—has no real solutions. Students identify these situations and make the appropriate geometric connections.
- Students define a complex number in the form  $a+bi$ , where  $a$  and  $b$  are real numbers and the imaginary unit  $i$  satisfies  $i^2 = -1$ . Students geometrically identify  $i$  as a multiplicand effecting a  $90^\circ$  counterclockwise rotation of the real number line. Students locate points corresponding to complex numbers in the complex plane.
- Students understand complex numbers as a superset of the real numbers; i.e., a complex number  $a+bi$  is real when  $b=0$ . Students learn that complex numbers share many similar properties of the real numbers: associative, commutative, distributive, addition/subtraction, multiplication, etc.
- Students solve quadratic equations with real coefficients that have complex solutions. They recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .
- Students solve quadratic equations with real coefficients that have complex solutions. Students extend polynomial identities to the complex numbers.
- Students note the difference between solutions to the equation and the x-intercepts of the graph of said equation.
- Students understand the Fundamental Theorem of Algebra; that all polynomial expressions factor into linear terms in the realm of complex numbers. Consequences, in particular, for quadratic and cubic equations are understood.

### Content map/calendar:

#### Lesson 1: Complex Numbers - Definition and Operations ( 3 days)

- Show graphically and analytically when an equation or systems of equations has no real solutions
- Develop  $i^2 = -1$ , define  $i = \sqrt{-1}$  and define complex number  $a+bi$  form
- Simplify powers of  $i$  and expressions involving exponents and powers of  $i$
- Complex number plane
- Addition, Subtraction, Multiplication with Complex numbers

#### Lesson 2: Complex Numbers as solutions to equations ( 2 days)

- Quadratic Formula and the discriminant

- examine all cases for the discriminant relate to graphs
- Solve Quadratic equations with complex roots
  - Complex roots come in conjugate pairs

### **Technology Component:**

Graphing calculator will be used to explore the behavior of polynomial functions.

### **Assessment:**

Students will be given formal and informal assessments through the course as the end of the unit.

## **Unit 6: Trigonometric Functions**

### **Student Outcomes:**

- Students apply geometric concepts in modeling situations. Specifically, they find distances between points of a circle and a given line to represent the height above the ground of a passenger car on a Ferris wheel as it is rotated a number of degrees about the origin from an initial reference point.
- Students sketch the graph of a nonlinear relationship between variables.
- Students model and graph two functions given by the location of a passenger car on a Ferris wheel as it is rotated a number of degrees about the origin from an initial reference position.
- Students define sine and cosine as functions for degrees of rotation of the ray formed by the positive x-axis up to one full turn.
- Students use special triangles to geometrically determine the values of sine and cosine for 30, 45, 60, and 90 degrees.
- Students define sine and cosine as functions for all real numbers measured in degrees.
- Students evaluate the sine and cosine functions at multiples of 30 and 45.
- Students define the tangent function and understand the historic reason for its name.
- Students use special triangles to determine geometrically the values of the tangent function for 30°, 45°, and 60°.
- Students define the secant function and the co-functions in terms of points on the unit circle. They relate these names for these functions to the geometric relationships among lines, angles, and right triangles in a unit circle diagram.
- Students use reciprocal relationships to relate the trigonometric functions and use these relationships to evaluate trigonometric functions for multiples of 30, 45, and 60 degrees.
- Students graph the sine and cosine functions and analyze the shape of these curves.
- For the sine and cosine functions, students sketch graphs showing key features, which include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity.
- Students explore horizontal scalings of the graph of  $y=\sin(x)$ .
- Students convert between degrees and radians.
- Students observe identities from graphs of sine and cosine basic trigonometric identities and relate those identities to periodicity, even and odd properties, intercepts, end behavior, and cosine as a horizontal translation of sine.
- Students formalize the periodicity, frequency, phase shift, midline, and amplitude of a general sinusoidal function by understanding how the parameters  $A$ ,  $\omega$ ,  $h$ , and  $k$  in the formula  $f(x) = A\sin(\omega(x-h)) + k$ .
- Students model cyclical phenomena from biological and physical science using trigonometric functions.
- Students understand that some periodic behavior is too complicated to be modeled by simple trigonometric functions.

- Students graph the tangent function.
- Students use the unit circle to express the values of the tangent function for  $\pi - x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of  $\tan(x)$ , where  $x$  is any real number in the domain of the tangent function.
- Students prove the Pythagorean identity  $\sin^2(x) + \cos^2(x) = 1$ .
- Students extend trigonometric identities to the real line, with attention to domain and range.
- Students use the Pythagorean identity to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$ , given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the terminal ray of the rotation.
- Students prove simple identities involving the sine function, cosine function, and secant function.
- Students recognize features of proofs of identities.
- Students see derivations and proofs of the addition and subtraction formulas for sine and cosine.
- Students prove some simple trigonometric identities.

## Content map/calendar:

### Lesson 1: Ferris Wheel - Tracking the Height and Co-height of a Passenger Car (1 day)

- Use a ferris wheel to model a trig function (sine functions model the height of a car on a ferris wheel).
- Discuss the domain and range of the functions.
- Discuss periodic functions and what other situations might be modeled by periodic functions.
- Using the ferris wheel to model the co-height (function that represents the horizontal displacement of the car from the center of the wheel, which is the cosine).
- Compare sine and cosine curves.

### Lesson 2: From Circle-ometry to Trigonometry (2 days)

- Formalize the idea of the height and co-height of a Ferris wheel and defined the sine and cosine functions that give the  $x$  and  $y$  coordinates of the intersection of the unit circle and the initial ray rotated through  $\theta$  degrees, for most values of  $\theta$  with  $0 < \theta < 360$ .
- The value of  $\cos(\theta)$  is the  $x$ -coordinate of the intersection point of the terminal ray and the unit circle.
- The value of  $\sin(\theta)$  is the  $y$ -coordinate of the intersection point of the terminal ray and the unit circle.
- The sine and cosine functions have domain of all real numbers and range  $[-1, 1]$ .

### Lesson 3: Extending the Domain of Sine and Cosine to all Real Number (1 day)

- Formalize the definition of the sine and cosine functions of a number of degrees of rotation,  $\theta$ . We rotate the initial ray made from the positive  $x$ -axis through  $\theta$  degrees, going

counterclockwise if  $\theta > 0$  and clockwise if  $\theta < 0$ . The point P is defined by the intersection of the terminal ray and the unit circle.

- The value of  $\cos(\theta)$  is the x-coordinate of P.
- The value of  $\sin(\theta)$  is the y-coordinate of P.
- The sine and cosine functions have domain of all real numbers and range  $[-1, 1]$ .

#### Lesson 4: Tangent (1 day)

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- The value of  $\tan(\theta)$  is the length of the line segment on the tangent line to the unit circle centered at the origin from the intersection with the unit circle and the intersection with the secant line created by the x-axis rotated  $\theta^\circ$  (this is why we call it tangent).
- The value of  $\tan(\theta)$  is the slope of the line obtained by rotating the x-axis  $\theta$  degrees about the origin.
- Domain, Range and Period of the tangent function.

#### Lesson 5: Secant and the Co-Functions (2 days)

- Reciprocal Functions
- Co-Functions

#### Lesson 6: Graphing Sine and Cosine Functions (1 day)

- Graph  $y = \sin x$  and  $y = \cos x$  and compare graphs
- Include discussion on amplitude, maximum, minimum, end behavior, odd vs. even function, period

#### Lesson 7: Radians and Degrees (2 days)

- Definition of a Radian - A radian is the measure of the central angle of a sector of a circle with arc length of one radius length.
- Converting from Degrees to Radians and vice versa.

#### Lesson 8: Trig Identities from Graphs (1 day)

- $\sin(x + 2\pi) = \sin(x)$
- $\cos(x + 2\pi) = \cos(x)$
- $\sin(x + \pi) = -\sin(x)$
- $\cos(x + \pi) = -\cos(x)$
- $\sin(-x) = -\sin(x)$
- $\cos(-x) = \cos(x)$



### Lesson 9: Transforming the graph of the Sine Functions (2 days)

- $f(x) = A\sin(\omega(x-h)) + k$
- $|A|$  = amplitude
- $\frac{2\pi}{|\omega|}$  = period
- $\frac{|\omega|}{2\pi}$  = frequency
- $h$  is called the phase shift.
- The graph of  $y = k$  is called the midline.

### Lesson 10: Using Trigonometric Functions to Model Cyclical Behavior (2 days)

- We can model periodic data with either a sine or a cosine function by extrapolating values of the parameters  $A$ ,  $\omega$ ,  $h$ , and  $k$  from the data and defining a function  $f(x) = A\sin(\omega(x-h)) + k$  or  $f(x) = A\cos(\omega(x-h)) + k$ , as appropriate.
- Sine or cosine functions may not perfectly fit most data sets from actual measurements; therefore, there are often multiple functions used to model a data set.
- If possible, plot the data together with the function that appears to fit the graph. If it is not a good fit, adjust the model and try again.

### Lesson 11: Graphing the Tangent Function (1 day)

- Graph  $y = \tan(x)$
- Determine trig identities from the tangent graph.

### Lesson 12: Trigonometric Identities (3 days)

- $\sin^2(\theta) + \cos^2\theta = 1$
- Prove simple identities involving the sine function, cosine function, and secant function.
- Prove the addition and subtraction formulas for sine and cosine.

### Technology Component:

Graphing calculator will be used to explore the behavior of trig functions.

### Assessment:

Students will be given formal and informal assessments through the course as the end of the unit.

## **Unit 7: Exponents, Exponential Equations, Logarithms, Log Equations**

## Student Outcomes:

- Students review and practice applying the properties of exponents for integer exponents.
- Students model a real-world scenario involving exponential growth and decay.
- Students review place value and scientific notation.
- Students use scientific notation to compute with large numbers.
- Students will calculate quantities that involve positive and negative rational exponents.
- Students rewrite expressions involving radicals and rational exponents using the properties of exponents.
- Students approximate the value of quantities that involve positive irrational exponents.
- Students extend the domain of the function  $f(x) = bx$  for positive real numbers  $b$  to all real numbers.
- Students write an exponential function that represents the amount of water in a tank after  $t$  seconds if the height of the water doubles every 10 seconds.
- Students discover Euler's number  $e$  by numerically approaching the constant for which the height of water in a tank equals the rate of change of the height of the water in the tank.
- Students calculate the average rate of change of a function.
- Students solve simple exponential equations numerically.
- Students use logarithms to determine how many characters are needed to generate unique identification numbers in different scenarios.
- Students understand that logarithms are useful when relating the number of digits in a number to the magnitude of the number and that base 10 logarithms are useful when measuring quantities that have a wide range of values such as the magnitude of earthquakes, volume of sound, and pH levels in chemistry.
- Students calculate a simple logarithm using the definition.
- Students construct a table of logarithms base 10 and observe patterns that indicate properties of logarithms.
- Students construct a table of logarithms base 10 and observe patterns that indicate properties of logarithms.
- Students justify properties of logarithms using the definition and properties already developed.
- Students understand how to change logarithms from one base to another.
- Students calculate logarithms with any base using a calculator that computes only logarithms base 10 and base  $e$ .
- Students justify properties of logarithms with any base.

- Students solve simple logarithmic equations using the definition of logarithm and logarithmic properties.
- Students use logarithm tables to calculate products and quotients of multi-digit numbers without technology.
- Students understand that logarithms were developed to speed up arithmetic calculations by reducing multiplication and division to the simpler operations of addition and subtraction.
- Students solve logarithmic equations of the form  $\log(X) = \log(Y)$  by equating  $X = Y$ .

## **Content map/calendar:**

### **Lesson 1: Integer Exponents ( 2 day )**

- Review Properties of Exponents
- Apply Properties of Exponents to rewrite expressions
- Model a real world scenario involving exponential growth and decay
- Review place Value and scientific notation
- Use scientific notation to compute large numbers

### **Lesson 2: Rational Exponents ( 2 day )**

- Extend properties of integer exponents to rational exponents
- Rewrite expressions with rational exponents using radicals and visa versa
- Use terminology “ nth root of a number”
- Discuss principal nth root
- Rewrite expressions involving radicals and rational exponents using properties of exponents

### **Lesson 3: Euler’s Number e: ( 1 day )**

- Use discovery lesson with exponential function and average rate of change to discover Euler’s number e
- Define formula to calculate average rate of change and use it

### **Lesson 4: Exponential Equations with common Bases ( 1 day )**

- Solve exponential equations with common bases algebraically

### **Lesson 5: Logarithmic ( What power) Function ( 1 day )**

- Develop definition of logarithm using “What power function”
- Evaluate basic log expressions

### **Lesson 6: Properties of Logarithms ( 3 days )**

- Define Log Rules Expand and contract log expressions
- Use Properties to solve real world situations exp equations with uncommon bases
- Justify Properties of Logarithms using the definition and properties already developed
- Real World Examples using logs
- Logs base 10

\*See module 3 lessons 9-13

### **Lesson 7: Changing the base ( 1 day )**

- Develop and use the change of base formula for logarithms

### **Lesson 8: Solving Logarithmic Equations (2 days)**

- Solve log equations
  - Single log on one side only
  - logs on both sides
  - Use rules to condense a side to a single logarithm before solving

\*Check for extraneous solutions

### **Technology Component:**

Graphing calculator will be used to explore the behavior of exponential and log functions.

### **Assessment:**

Students will be given formal and informal assessments through the course as the end of the unit.

## **Unit 8: Exponential and Logarithmic Functions and their Graphs**

### **Student Outcomes:**

- Students interpret addition and multiplication of two irrational numbers in the context of logarithms and find better-and-better decimal approximations of the

sum and product, respectively.

- Students work with and interpret logarithms with irrational values in preparation for graphing logarithmic functions.
- Students graph the functions  $f(x) = \log(x)$ ,  $g(x) = \log_2(x)$ , and  $h(x) = \ln(x)$  by hand and identify key features of the graphs of logarithmic functions.
- Students compare the graph of an exponential function to the graph of its corresponding logarithmic function.
- Students note the geometric relationship between the graph of an exponential function and the graph of its corresponding logarithmic function.
- Students will understand that the logarithm function base  $b$  and the exponential function base  $b$  are inverse functions.
- Students study transformations of the graphs of logarithmic functions.
- Students use the properties of logarithms and exponents to produce equivalent forms of exponential and logarithmic expressions. In particular, they notice that different types of transformations can produce the same graph due to these properties.
- Students understand that the change of base property allows us to write every logarithm function as a vertical scaling of a natural logarithm function.
- Students graph the natural logarithm function and understand its relationship to other base  $b$  logarithm functions. They apply transformations to sketch the graph of natural logarithm functions by hand.

## **Content map/calendar:**

### **Lesson 1 : Graphing The Logarithmic Function ; Graphs of Exponential Functions and Logarithmic Functions ( 2 day )**

- Graph several log functions by hand Include natural log function
- Discuss Domain, x-intercepts, y-intercepts, range
- Discuss behavior as  $x$  approaches 0 and as  $x$  approaches infinity
- Discuss Log graph is the reflection of its corresponding exponential graph in the line  $y=x$

### **Lesson 2 :Function Inverses ( 1 day )**

- Define inverse functions

- Write inverse equations for simple functions

### **Lesson 3: Relationship Between Logarithmic and Exponential Functions ( 1 day)**

- Develop logarithmic functions are inverse of exponential functions
- Solve exponential equations by using logs

### **Lesson 4 : Transformations of the Graphs of Logarithmic and Exponential Functions (1 day)**

- Study transformations of the graphs of logarithmic functions
- Use properties of logarithms and exponents to produce equivalent forms and see that graphs are equivalent
- use change of base to understand that every log function can be expressed as a vertical scaling of the natural log function

### **Lesson 5 : Modeling: Choose from linear, quadratic, sinusoidal, exponential and logarithmic functions (2 days)**

- Students analyze data and determine which type of function is best to model the data
- Use properties of exponents to interpret expressions for exponential functions

\*See lessons 22 and 23 for examples

### **Lesson 6: Solving Exponential Real World Problems ( 1 day)**

- Apply properties of logarithms to solve exponential equations
- Set each side of equation to  $=y$  and solve graphically as intersection of line and exponential graph

### **Technology Component:**

Graphing calculator will be used to explore the behavior of exponential and logarithmic functions.

### **Assessment:**

Students will be given formal and informal assessments through the course as the end of the unit.

## Unit 9 : Sequences

### Student Outcomes:

- Students use geometric sequences to model situations of exponential growth and decay.
- Students write geometric sequences explicitly and recursively and translate between the two forms.
- Students develop a general growth/decay rate formula in the context of compound interest.
- Students compute future values of investments with continually compounding interest rates.
- Students create exponential functions to model real-world situations.



- Students use logarithms to solve equations of the form  $f(t)=a \cdot b^{ct}$  for  $t$ .
- Students decide which type of model is appropriate by analyzing numerical or graphical data, verbal descriptions, and by comparing different data representations.
- Students apply knowledge of exponential and logarithmic functions and transformations of functions to a contextual situation.
- Students derive the sum of a finite geometric series formula.
- Students apply the sum of a finite geometric series formula to a structured savings plan.
- Students use the sum of a finite geometric series formula to develop a formula to calculate a payment plan for a car loan and use that calculation to derive the present value of an annuity formula.
- Students will compare payment strategies for a decreasing credit card balance.
- Students will apply the sum of a finite geometric series formula to a decreasing balance on a credit card.
- Students model the scenario of buying a house.
- Students recognize that a mortgage is mathematically equivalent to car loans studied in Lesson 30 and apply the present value of annuity formula to a new situation.
- Students use geometric series to calculate how much money should be saved each month to have 1 million in assets within a specified amount of time.

## **Content map/calendar:**

### **Lesson 1 : Geometric Sequences and exponential growth and decay (2 days)**

- Write explicit and recursive definitions for geometric sequences - translate between the two
- Use geometric sequences to model situations of exponential growth and decay
- 

### **Lesson 2: Compound Interest (1 day)**

- Develop a general growth/decay rate formula in the context of compound interest
- Use compound interest formula to compute future values of investments

### **Lesson 3: Modeling with Exponential Functions (1 day)**

- Real World Modeling with Exponential Functions see lessons 27 and 28

### **Lesson 4: Math of a saving plan 2 days)**

- Summation Notation
- Derive sum formula for geometric series
- Apply sum formulas to real world application

### **Lesson 5: Modeling and solving real world situations with sum of geometric sequence (4**

days)

- Use sum of geometric series to solve real world situations see lessons 30-33

### **Technology Component:**

Graphing calculator will be used to explore the behavior of exponential and logarithmic functions.

### **Assessment:**

Students will be given formal and informal assessments through the course as the end of the unit.

## **Unit 10: Probability and Normal Distribution**

### **Student Outcomes:**

- Students determine the sample space for a chance experiment.
- Given a description of a chance experiment and an event, students identify the subset of outcomes from the sample space corresponding to the complement of an event.
- Given a description of a chance experiment and two events, students identify the subset

of outcomes from the sample space corresponding to the union or intersection of two events.

- Students calculate the probability of events defined in terms of unions, intersections, and complements for a simple chance experiment with equally likely outcomes.
- Students calculate probabilities given a two-way table of data..
- Students interpret probabilities in context.
- Students construct a hypothetical 1000 two-way table from given probability information and use the table to calculate the probabilities of events.
- Students calculate conditional probabilities given a two-way data table or using a hypothetical 1000 two-way table.
- Students interpret probabilities, including conditional probabilities, in context.
- Students use two-way tables (data tables or hypothetical 1000 two-way tables) to determine if two events are independent.
- Students represent events by shading appropriate regions in a Venn diagram.
- Given a chance experiment with equally likely outcomes, students calculate counts and probabilities by adding or subtracting given counts or probabilities.
- Students use the complement rule to calculate the probability of the complement of an event and the multiplication rule for independent events to calculate the probability of the intersection of two independent events.
- Students recognize that two events A and B are independent if and only if  $P(A \text{ and } B) = P(A)P(B)$  and interpret independence of two events A and B as meaning that the conditional probability of A given B is equal to  $P(A)$ .

- Students use the formula for conditional probability to calculate conditional probabilities and interpret probabilities in context.
- Students use the addition rule to calculate the probability of a union of two events.
- Students describe data distributions in terms of shape, center, and variability.
- Students use the mean and standard deviation to describe center and variability for a data distribution that is approximately symmetric.
- Students draw a smooth curve that could be used as a model for a given data distribution.
- Students recognize when it is reasonable and when it is not reasonable to use a normal curve as a model for a given data distribution
- Students calculate z scores.
- Students use technology and tables to estimate the area under a normal curve.
- When appropriate, students select an appropriate normal distribution to serve as a model for a given data distribution.

## **Content map/calendar:**

### **Lesson 1: Chance Experiments, Sample Spaces, and events (1 day)**

- Define sample space and event for a chance experiment.
- Show that for a chance experiment in which outcomes of the sample space are equally likely, the probability of an event is the number of outcomes in the event divided by the number of outcomes in the sample space.
- Events can be described in terms of “or”, “and”, or “not”.

## **Lesson 2: Calculating Probabilities of Events Using Two-Way Tables (1 Day)**

- Use data organized in a two-way frequency table can to calculate probabilities.
- Interpret probabilities in context.

## **Lesson 3: Calculating Conditional Probabilities and Evaluating Independence Using Two-Way Tables (2 days)**

- Use data organized in a two-way frequency table to calculate conditional probabilities.
- Interpret probabilities in context.
- Define independent events a chance experiment.
- Use a two-way frequency table to determine if two events are independent or not independent.

## **Lesson 4: Events and Venn Diagrams (1 day)**

- Show students how to use Venn diagrams to represent probability events.
- Show students how to show combinations of events using “and,” “or,” and “not” by shading the appropriate regions of the Venn diagram.

## **Lesson 5: Probability Rules (2 days)**

- For any event A,  $P(\text{not } A) = 1 - P(A)$ .
- For any two events A and B,  $P(A \text{ given } B) = P(A \text{ and } B) / P(B)$ .
- Events A and B are independent if and only if  $P(A \text{ and } B) = P(A)P(B)$ .

- The addition rule states that for any two events A and B,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ .
- Two events are said to be disjoint if they have no outcomes in common. If A and B are disjoint events, then  $P(A \text{ or } B) = P(A) + P(B)$ .

### **Lesson 6: Distributions - Center, Shape, and Spread (1 day)**

- Distributions are described by the shape (symmetric or skewed), the center, and the spread (variability) of the distribution.
- A distribution that is approximately symmetric can take different forms.
- A distribution is described as mound shaped if it is approximately symmetric and has a single peak.
- A distribution is skewed to the right or skewed to the left if one of its tails is longer than the other.
- The mean of a distribution is interpreted as a typical value and is the average of the data values that make up the distribution.
- The standard deviation is a value that describes a typical distance from the mean.

### **Lesson 7: Using a Curve to Model a Data Distribution (1 day)**

- A normal curve is symmetric and bell shaped. The mean of a normal distribution is located in the center of the distribution. Areas under a normal curve can be used to estimate the proportion of the data values that fall within a given interval.
- When a distribution is skewed, it is not appropriate to model the data distribution with a

normal curve.

### **Lesson 8: Normal Distribution (2 days)**

- Show students how to find probabilities associated with normal distributions by finding the z-score (which measures how many standard deviations the value is from the mean) and then using the standard normal table.
- Show students how to find probabilities associated with normal distributions by using the calculator (no need to convert to z-score first).

### **Technology Component:**

Students will use graphing calculator to calculator probabilities under a normal curve.

### **Assessment:**

Students will be given formal and informal assessments through the course as the end of the unit.

## **Unit 11: Statistics**

### **Student Outcomes:**

- Students distinguish between observational studies, surveys, and experiments.
- Students explain why random selection is an important consideration in observational studies and surveys and why random assignment is an important consideration in experiments.
- Students recognize when it is reasonable to generalize the results of an observational

study or survey to some larger population and when it is reasonable to reach a cause-and-effect conclusion about the relationship between two variables.

- Students differentiate between a population and a sample.
- Students differentiate between a population characteristic and a sample statistic.
- Students recognize statistical questions that are answered by estimating a population mean or a population proportion.
- Students understand the term “sampling variability” in the context of estimating a population proportion.
- Students understand that the standard deviation of the sampling distribution of the sample proportion offers insight into the accuracy of the sample proportion as an estimate of the population proportion.
- Students use data from a random sample to estimate a population proportion.
- Students calculate and interpret margin of error in context.
- Students know the relationship between sample size and margin of error in the context of estimating a population proportion.
- Students understand the term “sampling variability” in the context of estimating a population mean.
- Students understand that the standard deviation of the sampling distribution of the sample mean offers insight into the accuracy of the sample mean as an estimate of the population mean.
- Students use data from a random sample to estimate a population mean.
- Students calculate and interpret margin of error in context.



- Students know the relationship between sample size and margin of error in the context of estimating a population mean.
- Students interpret margin of error from reports that appear in newspapers and other media.
- Students critique and evaluate statements in published reports that involve estimating a population proportion or a population mean.
- Given a description of a statistical experiment, students identify the response variable and the treatments.
- Students recognize the different purposes of random selection and of random assignment.
- Students recognize the importance of random assignment in statistical experiments.
- Students understand that when one group is randomly divided into two groups, the two groups' means will differ just by chance (a consequence of the random division).
- Students understand that when one group is randomly divided into two groups, the distribution of the difference in the two groups' means can be described in terms of shape, center, and spread.
- Given data from a statistical experiment with two treatments, students create a randomization distribution.
- Students use a randomization distribution to determine if there is a significant difference between two treatments.
- Students critique and evaluate statements in published reports that involve determining if there is a significant difference between two treatments in a statistical experiment.

## **Content map/calendar:**

### **Lesson 1: Types of Statistical Studies (1 day)**

- There are three major types of statistical studies: observational studies, surveys, and experiments.
  - An observational study records the values of variables for members of a sample.
  - A survey is a type of observational study that gathers data by asking people a number of questions.
  - An experiment assigns subjects to treatments for the purpose of seeing what effect the treatments have on some response.
- To avoid bias in observational studies and surveys, it is important to select subjects randomly.
- Cause and effect conclusions cannot be made in observational studies or surveys.
- In an experiment, it is important to assign subjects to treatments randomly in order to make cause-and-effect conclusions

### **Lesson 2: Using Sample Data to Estimate a Population (1 day)**

- We refer to summary measures calculated using data from an entire population as population characteristics. We refer to summary measures calculated using data from a sample as sample statistics. To generalize from a sample to the corresponding population, it is important that the sample be a random sample from the population. A random sample is one that is selected in a way that gives every different possible

sample an equal chance of being chosen.

### **Lesson 3: Sample Variability in the Sample Proportion (2 days)**

- The sampling distribution of the sample proportion can be approximated by a graph of the sample proportions for many different random samples. The mean of the sampling distribution of the sample proportions will be approximately equal to the value of the population proportion.
- As the sample size increases, the sampling variability in the sample proportion decreases – the standard deviation of the sampling distribution of the sample proportions decreases.

### **Lesson 4: Margin of Error When Estimating a Population Proportion (2 days)**

- Investigate how to make an inference about an unknown population proportion based on a random sample from that population.
- Show how random samples from populations with known proportions of successes behave by simulating sampling distributions for samples drawn from those populations.
- Compare an observed proportion of successes from a random sample drawn from a population with an unknown proportion of successes to these sampling distributions gives you some information about what populations might produce a random sample like the one you observed.
- These plausible population proportions can be described as  $p \pm M$ . Where

$M = 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ , where  $\hat{p}$  is the proportion of successes in a random sample of size  $n$ .

### **Lesson 5: Sampling Variability in the Sample Mean (2 Days)**

- For a given sample you can find the sample mean.
- There is variability in the sample mean. The value of the sample mean varies from one random sample to another.
- A graph of the distribution of sample means from many different random samples is a simulated sampling distribution.
- Sample means from random samples tend to cluster around the value of the population mean. That is, the simulated sampling distribution of the sample mean will be centered close to the value of the population mean.
- The variability in the sample mean decreases as the sample size increases.
- Most sample means are within two standard deviations of the mean of the simulated sampling distribution.

### **Lesson 6: Margin of Error When Estimating a Population Mean (2 days)**

- When using the sample mean to estimate a population mean, it is important to know something about how accurate that estimate might be.
- Accuracy can be described by the margin of error.
- The margin of error can be estimated using data from a single random sample (without

the need to create a simulated sampling distribution) by using the formula  $2\left(\frac{s}{\sqrt{n}}\right)$ , where  $s$  is the standard deviation of a single sample, and  $n$  is the sample size.

### **Lesson 7: Evaluating Reports Based on Data from a Sample (1 day)**

- It is important to interpret margin of error in context.
- It is unlikely that the estimate of a population proportion or mean will be farther from the actual population value than the margin of error.

### **Lesson 8: Experiments and the Role of Random Assignment (1 day)**

- Define includes experiment, subject, response variable, treatments, random selection, and random assignment.

### **Lesson 9: Differences Due to Random Assignment (1 day)**

- When a single set of values is randomly divided into two groups,
  - The two group means will tend to differ just by chance.
  - The distribution of random groups' means will be centered at the single set's mean.
  - The range of the distribution of the random groups' means will be smaller than the range of the data set.
  - The shape of the distribution of the random groups' means will be symmetrical.

### **Lesson 10: Ruling Out Chance and Drawing a Conclusion from an experiment (3 days)**

- The following are the general steps for carrying out a randomization test to analyze the results of an experiment.
- Step 1—Develop competing claims: No Difference vs. Difference
  - Develop the null hypothesis: This claim is that there is no difference between the two groups in the experiment.
  - Develop the alternative hypothesis: The competing claim is that there is a difference between the two groups. This difference could take the form of a "different from," "greater than," or "less than" statement depending on the purpose of the experiment and the claim being assessed.
- Step 2—Take measurements from each group, and calculate the value of the “diff” statistic from the experiment.
  - This is the observed "Diff" value from the experiment.
- Step 3—Randomly assign the observations to two groups, and calculate the difference between the group means. Repeat this several times, recording each difference.
  - This will create the randomization distribution for the "Diff" statistic under the assumption that there is no statistically significant difference between the two groups.
- Step 4—With reference to the randomization distribution (from Step 3) and the inequality in your alternative hypothesis (from Step 1), compute the probability of getting a "Diff" value as extreme as or more extreme than the "Diff" value you obtained in your experiment (from Step 2).
- Step 5—Make a conclusion in context based on the probability calculation (from Step

4).

- Small probability: If the "Diff" value from the experiment is unusual and not typical of chance behavior, your experiment's results probably did not happen by chance. The results probably occurred because of a statistically significant difference in the two groups.
- Not a small probability: If the "Diff" value from the experiment is NOT considered unusual and could be typical of chance behavior, your experiment's results may have just happened by chance and NOT because of a statistically significant difference in the two groups.

### **Lesson 11: Evaluating Reports Based on Data from and Experiment (1 day)**

- A cause and effect relationship can only be shown by a well-designed experiment.
- Randomly assigning the subjects to treatment groups evens out the effects of extraneous variables to create comparable treatment groups.
- A control group (which may be a placebo group) or a comparison group (a standard treatment) is sometimes included in an experiment so that you can evaluate the effect of the treatment.
- The number of subjects in each treatment (sample size) should be large enough for the random assignment to experimental groups to create groups with comparable variability between the subjects.

### **Technology Component:**

Using graphing calculator to determine if there is a significant difference between two treatments.

**Assessment:**

Students will be given formal and informal assessments through the course as the end of the unit.